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Implications of a simple competition model for the stability of an intercropping system

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Abstract

It is a common view that intercropping systems of agricultural crops produce more stable yields than do systems in which the same crops are grown in monoculture. This paper discusses a modelling approach which has been used to support the notion that whether or not intercropping is more stable than monoculture depends on the mode of interaction among crops, i.e. whether two different crops suppress or enhance each other. It is shown here that this notion is not supported by the model used. We conclude that the relative merits of the two cropping systems depend on the proportion of land allocated to each crop rather than on the mode of interaction. The model suggests that if the optimum allocation of land is considered, both systems will be equally stable.

Keywords: Agricultural ecosystems; Competition interspecific; Multispecies ecosystems; Stability

1. Introduction

In the context of crop research there is a growing interest in the subject of yielding stability. It is common to assess stability in terms of some measure of variability of yield. According to this concept, stability increases as variability of yield is reduced, where variability is defined by an appropriate measure of variability. Various such measures have been proposed, and several reviews on the subject are available (Goodman, 1975; Murdoch, 1975; Lin et al., 1986; Westcott, 1986; Becker and León, 1988).

In an analysis of cropping systems, one may study the comparative merits of different strategies (e.g. intercropping vs. monocropping; different allocation of land between crops) by means of an appropriate stability measure. There are several criteria for identifying optimal strategies, i.e. maximum mean of yield, minimum of yield variance, and minimum risk (Mead et al., 1986; Svirezhev and Racsko, 1992). In this paper we will focus on the criterion of a minimum coefficient of variation of yield.

It is often held that intercropping systems are better able to buffer environmental variability, i.e. they tend to be more stable than monocultures (see e.g. Reich and Atkins, 1970; Abalu, 1977; Francis and Sanders, 1978). This view has recently been questioned by Schulz (1984), Van-

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dermeer (1989), and Vandermeer and Schulz (1990) based on a simple competition model. Using the coefficient of variation and the ratio of range and mean as variability measures, Vandermeer (1989) concludes that whether or not intercropping is more stable than monoculture, depends on the mode of interaction among the crops grown together. He states that "intercrops will tend to be more variable than monocultures if competition is operative, that they may be either more or less variable when facilitation is operative, and that they will tend to be less variable in those rare cases where mutualism operates". Facilitation means that one crop is itself suppressed while enhancing the growth of the second crop. Mutualism refers to a situation where both crops grow better as intercrops than as monocrops.

It is held that this conclusion is in contrast to the underlying modelling approach. The purpose of this paper is to present a theoretical re-evaluation of the problem using the model of Vandermeer and Schulz (1990) and to draw conclusions regarding the stability of intercropping systems as compared to monoculture.

2. Modelling approach

Suppose that we wish to compare an intercropping system composed of two crops to a system where both crops are grown in monoculture. To do so, it is useful to consider four variables: monoculture yield of crop 1, monoculture yield of crop 2, yield in intercrop of crop 1, and yield in intercrop of crop 2. The interest is in the variability of the combination in intercrop as compared the variability in the system of both monocultures. To assess variability the yield trials are replicated in different environments (e.g. locations or years).

Vandermeer and Schulz (1990) suggest the coefficient of variation (CV), i.e. the standard deviation of yields in different environments as a proportion of the mean yield across environments, as a measure of stability. They propose to compare the CV of the sum of intercrop yields to the CV of the sum of monocrop yields. The CV will naturally depend on the fraction planted to both crops in both the intercropping and the monocropping system. If we assume that in the intercrop a fraction p is planted to crop 1 and the remainder $(1-p)$ is planted to crop 2, it has intuitive appeal to compare this to a monoculture system where the area is split according to exactly the same ratio, i.e. area of monocrop 1 : area of monocrop 2 $= p:(1-p)$.

Vandermeer and Schulz (1990) argue that this is not appropriate and that one should use optimal monoculture combinations when evaluating the stability (variability) of an intercrop, rather than fixing the monocultures at arbitrary proportions. According to Vandermeer and Schulz (1990) we ought to use that combination of monocultures which shows the lowest variability possible, if the goal is to determine whether the intercrop is less variable than monocultural alternatives. So the problem is to determine what exact p will give the minimum CV for the monoculture system. In what follows we present the solution given by Vandermeer and Schulz (1990). They define for the monoculture

x = yield of crop 1
y = yield of crop 2
μ_x = mean of crop 1
μ_y = mean of crop 2
V_x = variance of crop 1
V_y = variance of crop 2
$COV_{x,y}$ = covariance between crop 1 and 2
p = fraction of land planted to crop 1
$1-p$ = fraction of land planted to crop 2
CV_{x+y} = coefficient of variation of the total yield for the monocultural combination of both crops

Means μ_x and μ_y, variances V_x and V_y as well as the covariance $COV_{x,y}$ and the coefficient of variation CV_{x+y} are expected values for replications in different environments, i.e. they refer to the population of environments for which stability is to be assessed.

The variability of the sum of yields of crop 1 and 2 depends on the variances V_x and V_y as well as on the covariance $COV_{x,y}$ between the two crops. The covariance may be nonzero due to the influence of environmental factors. For example, an improvement of nutrient supply is likely to positively affect both crops, thus causing a positive covariance.

The contribution of crop 1 to the yield on the total area will equal p times x, the yield per area planted to crop 1. The total yield becomes $px + (1-p)y$. In order to compute CV_{x+y}, we therefore must evaluate mean and variance of the term $px + (1-p)y$.

We have

$$CV_{x+y} = \frac{\left[p^2V_x + (1-p)^2V_y + 2p(1-p)COV_{x,y}\right]^{1/2}}{p\mu_x + (1-p)\mu_y}. \quad (1)$$

Now we need to find the value of p which minimizes CV_{x+y}. Differentiating Eq. 1 with respect to p, setting the derivative equal to zero, and solving for p, one finds:

$$p^o = \frac{\mu_x V_x - \mu_y COV_{x,y}}{\mu_y V_x + \mu_x V_y - (\mu_x + \mu_y)COV_{x,y}}. \quad (2)$$

p^o gives the least variable combination for the monocrop system. Note that p^o may take on values less than zero or greater than one, since the function is defined beyond the range we are interested in. When this occurs the following interpretation is appropriate: if $p^o > 1$ then 100% of crop 1 is the least variable combination; if $p^o < 0$, then 100% of crop 2 is least variable.

We wish to stress here that the analysis by Vandermeer and Schulz (1990) is biased in favor of monoculture in that it requires an optimum combination of crop 1 and 2 for the monocultural system, but not for the intercropping system. Contrary to this treatment, we suggest to compute CV for the intercrop in the same way as for the monocrop, i.e. based on the optimum rather than on an arbitrary value of p.

To compare variabilities, Vandermeer and Schulz (1990) used a graphical method based on the ratio of range and median. Variability of a system is basically approximated by the product of the standardized ranges of both components. One flaw of this approach is that it ignores the covariance between the two crops, which will usually be different for the intercrop and the monoculture (see below). We will not discuss this graphical method further, but present an exact formula for CV of the intercrop, which greatly simplifies the analysis. We will do this by including the competition model used by Vandermeer and Schulz (1990).

The competition functions are given by

$$y^* = (1-p^*)y - ax^* \quad (3)$$

$$x^* = p^*x - by^* \quad (4)$$

where x^* and y^* are the intercrop yields of crop 1 and 2, a and b are the competition coefficients, and p^* is the fraction of land planted to crop 1 in the intercropping system (x and y are the monoculture yields of crop 1 and 2). If the competition coefficient a is positive, the competition of crop 1 reduces the yield of crop 2 compared to what would be expected in monoculture. If the coefficient a is negative, the growth of crop 2 is facilitated by the presence of crop 1. Based on the signs of a and b in Eqs. 3 and 4 we can distinguish four different cases:

$a,b > 0$: competition
$a > 0$ and $b < 0$: facilitation of crop 1
$a < 0$ and $b > 0$: facilitation of crop 2
$a,b < 0$: mutualism (mutual facilitation)

The notation in Eqs. 3 and 4 differs slightly from that in Vandermeer and Schulz (1990), but the meaning is the same. The change in notation is necessary since, for a reason which will become apparent later, we wish to express monocultural yields x and y in terms of the area to which the individual crop is planted, while intercrop yields x^* and y^* are expressed in terms of the whole area planted to both crops.

Solving the equation system given by Eqs. 3 and 4 yields

$$x^* = \frac{px - b(1-p)y}{1-ab}$$

and

$$y^* = \frac{(1-p)y - apx}{1-ab}.$$

Means, variances and covariances of x^* and y^* can be computed as

$$\mu_x^* = (1-ab)^{-1}\left[p^*\mu_x - b(1-p^*)\mu_y\right],$$

$$\mu_y^* = (1-ab)^{-1}\left[(1-p^*)\mu_y - ap^*\mu_x\right],$$

$$V_x^* = (1-ab)^{-2}\left[p^{*2}V_x + b^2(1-p^*)^2V_y - 2bp^*(1-p^*)COV_{x,y}\right],$$

$$V_y^* = (1-ab)^{-2}\left[(1-p^*)^2V_y + a^2p^{*2}V_x - 2ap^*(1-p^*)COV_{x,y}\right],$$

$$COV_{x^*,y^*} = (1-ab)^{-2}\left[-ap^{*2}V_x - b(1-p^*)^2V_y + (1+ab)p^*(1-p^*)COV_{x,y}\right].$$

Based on these equations we find the coefficient of variation for the intercrop to be:

$$CV_{x^*+y^*} = \left[(1-a)^2p^{*2}V_x + (1-b)^2(1-p^*)^2V_y + 2(1-a)(1-b)p^*(1-p^*)COV_{x,y}\right]^{1/2} \times \left[(1-a)p^*\mu_x + (1-b)(1-p^*)\mu_y\right]^{-1} \quad (5)$$

We observe that, provided the proportions p and p^* are chosen equal and competition is absent (i.e. $a = b = 0$), $CV_{x+y} = CV_{x^*+y^*}$. This result is to be expected, since if the crops do not interact in any way it theoretically does not make any difference whether they are grown as intercrops or on two separate parts of the growing area.

To find the optimum fraction p^{*o} for the intercrop, we differentiate with respect to p^* and set equal to zero in much the same way as for the monocrop (see Vandermeer, 1989). The result is

$$p^{*o} = \frac{(1-b)(\mu_xV_y - \mu_yCOV_{x,y})}{(1-a)(\mu_yV_x - \mu_xCOV_{x,y}) + (1-b)(\mu_xV_y - \mu_yCOV_{x,y})}$$

Again this is the same fraction as for the monocrop if competition is absent, i.e. if $a = b = 0$.

For a comparison of the stability of monocropping and intercropping, it is reasonable to determine the minimum coefficient of variation for each system. If this is done, the striking result is that the minimum coefficient of variation will always be the same in monocrop and intercrop, regardless of the values of the competition coefficients a and b! To see this, we make the following simplification:

$$t = (1-b)/(1-a)$$
$$u_1 = \mu_xV_y - \mu_yCOV_{x,y}$$
$$u_2 = \mu_yV_x - \mu_xCOV_{x,y}$$

which allows us to re-express p^{*o} and $1-p^{*o}$ as

$$p^{*o} = \frac{tu_1}{u_2 + tu_1} \quad \text{and}$$
$$1 - p^{*o} = \frac{u_2}{u_2 + tu_1}$$

Inserting these equations into Eq. 5, the coefficient of variation for the optimum fraction p^{*o} can then be rewritten as

$$CV^o_{x^*+y^*} = \frac{\left[u_1V_x + u_2V_y + 2u_1u_2COV_{x,y}\right]^{1/2}}{u_1\mu_x + u_2\mu_y} \quad (6)$$

As can be seen, t cancels out, and therefore $CV^o_{x^*+y^*}$ is independent of a and b as stated. In other words it is always true that $CV^o_{x^*+y^*} = CV^o_{x+y}$ (where CV^o_{x+y} is the minimum CV for the monocrop).

The conclusion is that in terms of the minimum coefficient of variation, monocropping and intercropping are equally stable, regardless of the type of competition in the intercrop, provided that the linear competition model in Eqs. 3 and 4 is valid. This is in contradiction to the interpretation by Vandermeer (1989) and Vandermeer and Schulz (1990).

3. Another look at stability

In practice, it will be difficult if not impossible to determine the optimal p for both monocrop and intercrop. But we may expect that the experienced farmer will be more or less close to the optimal fraction. To compare stability we might then ask, how the two systems behave in the vicinity of the optimal p. It would be desirable for the CV to change only slowly as p moves away from its optimum.

To analyse the stability for non-optimal p^*, one has to evaluate the functions $CV_{x+y} = f(p)$

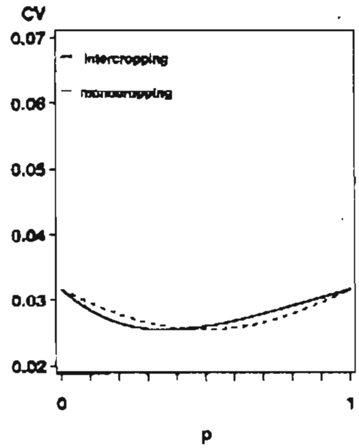

Fig. 1. Plot of coefficient of variation versus p for monocropping and intercropping; $\mu_x = \mu_y = 100$; $V_x = V_y = 10$; $COV_{x,y} = 3$; $a = 0.3$; $b = 0.6$.

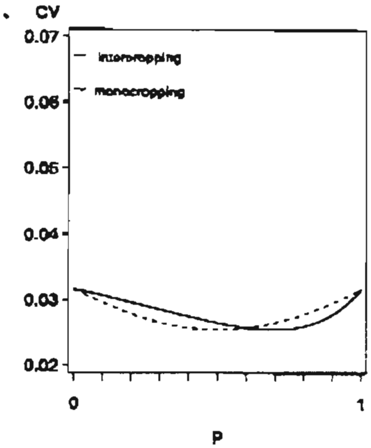

Fig. 2. Plot of coefficient of variation versus p for monocropping and intercropping; $\mu_x = \mu_y = 100$; $V_x = V_y = 10$; $COV_{x,y} = 3$; $a = 0.3$; $b = -0.6$.

and $CV_{x^*+y^*} = f(p^*)$ given by Eqs. 1 and 5. To assess the change in $CV_{x^*+y^*}$, we might look at the slope of $f(p^*)$ in the vicinity of p^{*o}. Unfortunately the explicit expressions for the first derivative with respect to p^* is not particularly simple and it depends on seven parameters, so an analytical treatment of the problem does not appear to be a straightforward exercise.

To shed some light on the problem, it seems instructing to inspect graphs $CV_{x+y} = f(p)$ and $CV_{x^*+y^*} = f(p^*)$ for some selected sets of parameter values. We present graphs for three such sets: Fig. 1:

$\mu_x = \mu_y = 100$; $V_x = V_y = 10$; $COV_{x,y} = 3$;
$a = 0.3$; $b = 0.6$

Fig. 2:

$\mu_x = \mu_y = 100$; $V_x = V_y = 10$; $COV_{x,y} = 3$;
$a = 0.3$; $b = -0.6$

Fig. 3:

$\mu_x = 50$; $\mu_y = 100$; $V_x = V_y = 10$; $COV_{x,y} = 0$;
$a = 0.3$; $b = -0.6$

In Figs. 1 to 3, $f(p)$ and $f(p^*)$ meet at $p = 0$ and $p = 1$. This will generally be so, also for other sets of parameter values, as can be seen from Eqs. 1 and 5. We find that $CV_{x^*+y^*} = CV_{x+y}$ for $p = 0$ and for $p = 1$.

We already know from Eq. 6 that one always has $CV^o_{x^*+y^*} = CV^o_{x+y}$, usually with different optimal values p^o and p^{*o}. This is also visible in Figs. 1 to 3. For $p^o > p^{*o}$, the monocrop will always show the same change in CV in the interval $[p^o,1]$ as does the intercrop in the interval $[p^{*o},1]$ (see also Figs. 2 and 3). Since $[p^o,1]$ is shorter than $[p^{*o},1]$ for $p^o > p^{*o}$, we conjecture that the monocrop becomes less stable more rapidly than the intercrop as p and p^* move away from the optimum towards a value of unity. On the contrary, the monocrop is better buffered as p and p^* change in the direction of zero. The situation is reversed if $p^o < p^{*o}$ (see Fig. 1).

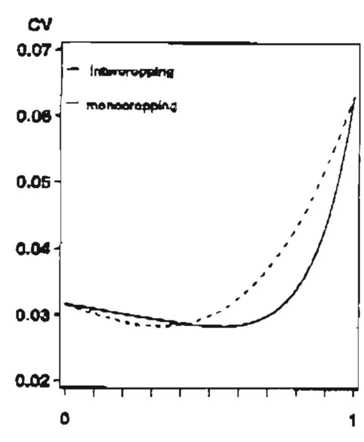

Fig. 3. Plot of coefficient of variation versus p for monocropping and intercropping; $\mu_x = 50$; $\mu_y = 100$; $V_x = V_y = 10$; $COV_{x,y} = 0$; $a = 0.3$; $b = -0.6$.

4. Conclusion

The analysis of the competition model used by Vandermeer and Schulz (1990) reveals that intercropping and monocropping are equally stable (as measured by the coefficient of variation), if optimum land area allocation is considered, i.e. if in both systems the allocation of area to crops 1 and 2 is such that variability of the total yield is minimized. In both systems, variability will increase (stability will decrease) as we depart from the optimum area allocation. The increase of variability in the intercrop may be more drastic or less drastic than for the monocrop, depending on the direction of departure and the values of the mean yields μ_x and μ_y, the variances and covariances V_x, V_y, and $COV_{x,y}$, as well as the competition coefficients a and b.

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