CONTINUOUS VERSUS DISCRETE ECONOMIC ANALYSIS OF EXPERIMENTAL DATA

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1. Introduction

Economists are often called upon to analyse experimental data in order to formulate appropriate recommendations for farmers. The data may be analysed as a set of discrete points or it may be possible to represent the data by a continuous response function. Some types of experiments, comparing different production processes, can only be analysed as a set of discrete points. An example is the comparison of chemical weed control with farmers' hand weed control methods. In this case we want to compare two discrete technologies for which there are no reasonable intermediate points. The analysis can usually be efficiently made by partial budgeting in which changes in revenues are compared with change in costs; considering only changes brought about by using chemical weed control. In more complex farming systems it may be useful sometimes to extend this analysis to a linear programming approach in which the changes in resources of labor and cash are evaluated in relation to farmers resource constraints and opportunities in other farm and nonfarm enterprises (e.g. Lynam and Sanders, 1980). However, the choice is still made among a discrete number of alternatives.

Fertilizer experiments and other experiments comparing different levels of a factor present the opportunity to use regression analysis to fit a continuous response function to the data and then by setting the marginal value(s) product of the input(s) equal to the input price(s), the profit maximizing level of the input(s) can be determined. By this method the optimum fertilizer level may be chosen from any one of an infinite number of points on the response curve and will not usually be one of the points represented by the treatments in the fertilizer experiments. On the other hand, partial budgeting methods may be used to select among a discrete number of treatments represented in the experiment, that which gives highest profits.[1/]

The purpose of this note is to discuss the advantages and disadvantages of continuous versus discrete analysis of fertilizer experimental data and in particular to relate one form of discrete analysis used by Perrin *et al*

1/ For purpose of this note we assume only the profit maximizing objective of farmers. We recognize other objectives, particularly risk aversions.

to the commonly employed production economics theory. We begin then with the analysis of a fertilizer experimental data set using a conventional response function. We extend this analysis to examine the sensitivity of the optimum fertilizer level and associated profits to changes in the level and cost of capital. We then turn to discrete analysis using partial budgeting and show how this approach can also be extended to examine the sensitivity of the optimal fertilizer levels and profits to changes in the cost and availability of capital.

2. Continuous Analysis with a Response Function

The standard response function analysis can be depicted as follows: Given a response function, $Y = F(x_i)$, i=1, ---N, where Y is output and x_i are levels of fertilizer nutrients, i, profit maximizing fertilizer levels are given by the solution of the N equations.

$$\frac{\partial Y}{\partial X_i} = r_i \qquad i=1, ---N$$

where r_i is the ratio of the price of nutrient i to the price of grain.

The task of the analyst is then to choose an appropriate functional form for the response function and appropriate assumptions about price ratios. Although there is a rich literature on alternative functional forms most applications (especially in LDC's) have gravitated to the standard quadratic form:

$$Y = a + bx_i + cx_1^2, + dx_2 + ex_2^2 + fx_1, x_2 - - -,$$

or to the square root form favored by Colwell (1979),

$$Y = a + bx_1 + cx_1^{0.5} + dx_2 + ex_2^{0.5} + fx_1^{0.5}x_2^{0.5}$$

Both have the advantage of ease of computation of the optimal levels of nutrients.[1/]

In choosing price ratios most analyst simply choose market prices of

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1/ The solution for the quadratic is, $x_1 = (fd - r_2f + 2eb)/(4ec - f^2)$, $x_2 = (r_1 - b - 2cx_1)/f$, and for the square root.
$\sqrt{x_1} = (2cd - 2cr_2 - fe)/(f^2 - 4r_1r_2 + 4br_2 + 4dr_1 - 4db)$, and $\sqrt{x_2} = (2\, x_1r_1 - 2\sqrt{x_1}b - c)/f$.

nutrients of grain and ignore several important farmer costs associated with using fertilizer.[1/] These are discussed in detail in Perrin et al (1976) and include costs of labor and equipment to apply the fertilizer, costs of capital tied up in fertilizer expenditures, and costs of harvesting the additional yield. Moreover, farmer yields for the same fertilizer treatment will typically be lower than experimental yields. Byerlee and Harrington (1979) show that inclusion of these additional considerations to better represent farmers' circumstances can reduce optimum fertilizer levels by half when compared with "naive" application of market prices.

Let us look at an example. Using the yield data in Table 1 from Perrin et al (1976) for maize under various levels of nitrogen, N, and Phosphorous, P, square root and quadratic functional forms were fitted to the average yields for each level of N, and P, to give the following functions (with standard errors of coefficients in brackets);

$$Y = \underset{(103)}{2{,}194} - \underset{(2.06)}{0.8467}N + \underset{(26.7)}{162.0}\sqrt{N} - \underset{(6.68)}{3.389}P + \underset{(48.9)}{55.36}\sqrt{P} + \underset{(2.31)}{10.38}\sqrt{NP}$$

$$R^2 = .994$$

$$Y = \underset{(109)}{2{,}129} + \underset{(2.47)}{27.9}N - \underset{(0.015)}{0.1033}N^2 + \underset{(7.02)}{21.84}P - \underset{(0.126)}{0.308}P^2 + \underset{(.0326)}{0.1168}NP$$

$$R^2 = .991$$

The relevant price ratio r_i, was calculated from the relation;

$$r_i = \frac{(1 + R)\ (p_i + T + L)}{C(p_y - H)}$$

where R is the cost of capital, p_i is the price of the nutrient, T is cost of transport (per unit of nutrient), L is cost of application, C is the ratio of farmers' yields to experimental yields, p_y is the price of grain and H is cost of harvesting, shelling and transporting maize. Following the example in Perrin et al these values were set as follows: $p_N = 5$, $p_P = 8$, $T = 3$, $L = .07$ (on average)[2/], $p_y = 1.2$, $H = 0.2$ all expressed in \$/kg.

1/ Some additional benefits are also typically ignored especially added crop crop residues produced by fertilizer use,

2/ In practice, labor for fertilizer application does not vary proportionally with quantity applied but rather varies with the number of applications.

R was taken to be 0.40 (i.e. 40 percent interest rate) and C was assumed to be 0.9 (i.e. farmers yields 10 percent less than experimental yields). That is $r_n = 13.5$ and $r_p = 16.6$.

Using these ratios, the calculated optimum rates of N and P are:

Function	N (kg/ha)	P (kg/ha)
Quadratic	83	24
Square Root	46	10

Despite the good fit of both the quadratic and square root forms and the expected signs for all coefficients, there is a substantial difference in the estimated optimum for each form. On theoretical grounds there is no reason to choose one form over the other. However, the square root form does give a slightly better R^2, and in particular provides a better fit to the experiment points about the economic optima, i.e. 50-0, 50-25, 100-25, and 100-50 respectively of N and P.[1/] Hence, following Colwell (1978), we choose the square root form as a better representation of the data actually obtained.

These results alert us to some practical problems in continuous analysis of fertilizer data. The substantial difference in economic optima obtained with each method is common (e.g. Joly, 1976). While some analysts try various functional forms they usually base their choice on the R^2 measure of goodness of fit but do not consider the goodness of fit to the economically relevant points nor do they usually examine the sensitivity of the economic optima to the choice of functional form. Although, as Colwell (1978) notes the level of profits is fairly insensitive around the economic optimum, we know that risks will usually be higher for higher levels of expenditure and it is important to check that the optimal fertilizer level is not being inflated by the functional form chosen,

3. Exploring the Sensitivity of the Optimum Fertilizer Level to the Cost and Level of Capital Employed.

The above analysis has focused on finding a unique optimum solution. However, it is often a valuable exercise once a suitable function is fitted,

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[1/] The sum of squares of residuals about these four points is 64 percent greater for the quadratic than the square root form.

to explore the sensitivity of this optimum to the cost and level of capital employed.

When deriving recommendations for small farmers in LDC's, it is often particularly difficult to value the farmers' own resources of land, labor and capital since a well established market for these resources may not exist. Weed control recommendations may be very dependent on the value assigned to family labor (Harrington). In our case, optimum fertilizer levels are sensitive to the level and cost of operating capital employed. Some farmers, unable to borrow in the formal or informal markets, may face a rigid capital constraint. For others the return on capital necessary to induce farmers to invest scarce cash resources may be high. Interest rates on loans in the informal capital market are often in the order of 50 to 100 percent per annum and reflect this capital scarcity.

Sensitivity analysis to check the robustness of the economic optima to changes in level and cost of capital then provides the analyst with additional information on which to base his recommendation. In fact, researchers often find that the profit curve is relatively flat around the economic optimum and that with substantially less investment in fertilizer, almost the same level of profits can be obtained. Said another way the marginal return on additional capital expended is relatively low around the optimum.

The expansion path of response analysis is a useful tool for exploring the effect of the level and cost of capital on optimum fertilizer levels and profits. It shows the optimal level of nutrients for a given expenditure. The marginal return to additional capital expenditures can also be derived for each level of capital expended.

Given a response function $Y = f(x_1, x_2)$ the expansion path, following Dillon (1977), is given by the solution of the equation.[1/]

1/ These equations obtained by inclusion of a Lagrangian constraint for capital investment.

$$\frac{\partial y/\partial x_i}{\partial y/\partial x_2} = \frac{p_2^{\,a}}{p_1^{\,a}} \qquad\qquad p_1\, x_1 + p_2\, x_2 = \bar{C}$$

where $\bar{C}$ is the fixed capital outlay and $p_i^{\,a}$ are costs of employing one unit of nutrient, i. At any point on the expansion path the marginal return on capital, MRR (or shadow price) is given by:

$$\text{MRR} = p_y\,(\partial y/\partial x_i)/p_i^{\,a} - 1$$

which is the slope, $\partial\pi/\partial C$, along the expansion path showing changes in profits, for a change in expenditure, C.

In the arithmetic solution of these equations, the cost of capital may be included in the adjusted price $p_i^{\,a}$ of input x_i as before. That is,

$$p_i^{\,a} = (1 + R)\,(p_i + T + L)$$

In this case the MRR on capital is the marginal rate of return above the estimated cost of capital, R. More commonly, (Colwell (1978), Dillon (1977) we exclude the cost of capital in $p_i^{\,a}$ and the MRR is the marginal rate of return to the last unit of capital invested and can be compared to a range of estimates of the actual cost of capital, R.[1/] Profits by this method do not include the cost of capital. This is the method followed here:

Example

Using our preferred square root functional form fitted to the previous example, the expansion path, the marginal rate of return and profits at each point on the expansion path were computed and are shown by the solid lines in Figure 1. Notice that the optimum fertilizer levels are quite sensitive - but profits are relatively insensitve to the rate or return on capital, especially at low rates of return on capital. Assuming zero costs of capital (as is naively done in many studies), the optimal levels of N and P are 133 and 36 kg/ha respectively. But this increases expenditures to over double the cost for our optimum solution of 46N, 10P with cost of capital at 40 percent. At the same time, the solution 46N, 10P yields profits

[1/] This is of course equivalent to the shadow price in linear programming.

(compared to zero fertilizer use) of \$760/ha compared to \$900/ha for the solution 133N, 36P. If we use a minimum rate of return on capital of 100 percent costs are reduced to only half our optimum (at a return of 40 percent) but profits only fall to \$560.

All of this is of course nothing more than the familiar law of diminishing returns but it does demonstrate the additional information that can be obtained from the expansion curve. Another example is given in Figure 2 for the expansion path for a quadratic response function applied to wheat fertilization in Nepal (Flinn, 1979). This shows that initial capital should be spent on nitrogen fertilizer and potassium only added after 600 rupees of nitrogen had been purchased. An extension agent may then want to demonstrate two levels of fertilizer expenditure 600-rupees or about 100 kg N/ha and 1100 rupees with 150 kg N and 40 kg k/ha.

4. Discrete Analysis

Partial budgeting can be used to choose among the various treatment levels represented by the experimental data. Conventionally this would be performed by comparing added revenues and added costs to the check treatment and then choosing that treatment with the highest added profits. Table 2.1 shows such a calculation of two treatments of the Perrin *et al* example previously discussed. Clearly 50N and 25P is the preferred treatment in this set.

With many treatments it is computationally easier to calculate "net benefits" as gross revenues (or benefits) minus costs that vary for each treatment (including capital costs) as shown in Table 3.[1/] The term "net benefits" is used since its absolute value has no meaning in terms of profits becuase fixed costs are excluded. The treatment with the highest net benefit is the chosen - that is, the treatment 50N - 25P in Table 3.

Note that the treatment selected by discrete analysis (50N - 25P) is close to optimum (46N 10P) using the same cost and price assumption selected after a careful fitting of a continuous response function. Our experience has shown that this agreement in the outcome of discrete and continuous analysis is the rule rather than the exception (see Byerlee and Harrington, 1979) for another example).

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5. Extension of the Partial Budget Approach

As in the continuous analysis, it is useful to explore the sensitivity of the economic optimum to the cost and level of capital expended.

Let us assume for purposes of arguement that we have the previous fertilizer example represented by experimental data with treatments in one kilogram increments, i.e. 150 levels of N and 50 levels of P, i.e. an improbable 7500 treatment. The procedure for discrete analysis of the data to construct the expansion path and the profits or net benefits curve is developed by Perrin et al and consists of the following steps;

a) Compute costs, gross genefits and net benefits for each treatment. As with the continuous analysis costs and net benefits would not include capital costs.

b) Conduct dominance analysis on the array of treatment to obtain the expansion path. Strictly speaking this dominance analysis would eliminate any treatment whose costs were higher and yields equal to or lower than any other treatment. An even stronger criteria for dominance analysis is to eliminate all treatments whose costs are higher and net benefits equal to or lower than any other treatment.[1/] The resulting undominated treatments are then the treatments along the expansion path.

c) The net benefits and costs for the undominated treatments can then be plotted to obtain the net benefits curve. With our imaginary 7500 treatments, this curve will be practically the same as the net benefits curve obtained in Figure 1. The marginal rate of return on capital can then be calculated as the slope of this curve and interpreted in the same manner as for the continuous analysis.

In practice of course we conduct discrete analysis of experiments with a few treatments. Nonetheless the same procedure can be applied and the resulting undominated treatments and net benefit curve is the best approximation to the expansion path and net benefit curve allowed by the points in the experiment.

[1/] The undominated treatments for the second criteria - net benefits - are a subset of the treatments obtained from using yields for dominance analysis.

Returning to the experimental data of Table 1 and using the same cost and price assumptions, we obtain the net benefit curve and marginal rates of return on capital for the discrete case represented by the dotted line in Figure 1. Although this curve lacks the consistently declining return on capital of the continuous case (because the treatment points approximate the true expansion path to varying degrees) it is a useful approximation of the curve obtained from continuous analysis. It is in fact exactly analoguous to the curve and shadow price obtained by parametric linear programming as one resource constraint is varied.

Again the construction of the net benefit curve and the calculation the MRR on capital provides us with substantially more information than obtained from a simple partial budget which chooses the point with the highest net benefits (after subtracting capital costs). We see that with expenditure limited to less than $500/ha, farmers should allocate this all to purchase 50 kg N/ha. (The continuous analysis indicates 40 kg N/ha and 8 kg N/ha for the same expenditure). With unlimited capital available at a cost of 40 percent as we assumed earlier, the optimum is 50-25. However net benefits increase very little with further expenditures on fertilizer and it is unlikely even with lower costs of capital that farmers would choose higher fertilizer levels.

Finally, the construction of a net benefit curve provides the analyst with the first approximation of the expansion path followed with changing cost and level of capital. Moreover, the expansion path traces out the path of optimum fertilizer levels for changes in input-output price ratios (Input price ratios kept constant). This information can be used by researchers to design more efficient experiments which include more treatments along the path and eliminate treatments which are clearly irrelevant given reasonable assumptions on price ranges. In this case the treatments that add phosphate without nitrogen or which have high levels of nitrogen without phosphorous are identified as "outliers" which could be eliminated from further experimentation.

Conclusions

What then are the relative merits of discrete versus continuous analysis of experimental data. First we have noted from the outset that many experimental data sets comparing different techniques (e.g. hand weeding and herbicides) can only be analysed by discrete analysis. Our comments then focus on experimental data, largely fertilizer data, for which there is a choice of analytical technique.

Continuous analysis, when care used in fitting the functional form and specifying cost, price, and yield assumptions clearly will provide a more "precise" estimate of the economic optimum and the expansion path than discrete analysis of the same data. However, in LDC's with limited research resources we seek recommendations for broad groups of farmer and such precision is not warranted. Each farmer will have to adjust general recommendations to fit his own natural and economic circumstances. For example, we might be interested in knowing whether 0,50,100 or 150 kg of N should be applied and if phosphorous may be needed. Our experience has shown that discrete analysis usually gives results within a few kilograms from that obtained in the continuous analysis.

Moreover, continuous analysis has some drawbacks. Care must be used in ensuring that the fitted function adequately represents the data points, particularly those around the expansion path, otherwise results may be misleading. Also computer facilities are usually needed to run the regression. Finally, even using standard polynamial functions, computation of economic optima and particularly the expansion path require mathematical skills that in our experience present difficulties - even to economists. Therefore, computer programs such as those developed by Colwell (1978) may be needed for this purpose as well.

Finally, for obtaining relevant recommendations for farmers, we feel that the methodology used for the analysis (discrete or continuous) is secondary to the experimental design and the price assumptions used in the analysis. Our impression is that most fertilizer experiments are being conducted in agro-climatic conditions that do not represent target farmers, under practices (such as special tillage or herbicides) that farmers are not using and would not find profitable to use. Moreover choice of treatment usually reflect tradition rather than an attempt to represent treatments in the economically relevant range.

In the analysis of the experimental data, many costs are ignored and resulting recommendations are unreasonably high. Many economists using response analysis use simple market price ratios. Most analysts ignore harvesting costs, use unadjusted experimental yields and use bank rates of interest for the cost of capital if indeed cost of capital is included.

TABLE 1. AVERAGE MAIZE YIELDS FOR VARIOUS FERTILIZER TREATMENTS

	Nitrogen (kg/ha)			
	0	50	100	150
	(Yield, ton/ha)			
Phosphate (kg/ha) 0	2.21	3.14	3.91	4.01
25	2.44	3.88	4.40	4.84
50	2.36	4.05	4.74	5.16

Source: Perrin *et al* (1976); Table 2.

TABLE 2. EXPANSION PATH FOR SQUARE RATE RESPONSE FUNCTION

Capital Expenditures ($/ha)	Optimal Level[1/] (kg/ha) N	P	Yield	MRR on Capital (%)
0	0	0	2190	
86	8	1	2747	200
214	20	4	3085	100
313	29	6	3287	70
424	39	8	3483	50
503	46	10	3611	40
607	55	12	3767	30
748	67	16	3962	20
944	83	21	4212	10
1073	94	24	4366	5
1231	107	28	4546	0

[a/] Calculated for $r_n = 9.7$, $r_p = 11.9$

TABLE 3. PARTIAL BUDGET OF AVERAGE DATA FROM FERTILIZER TRIALS (PER HECTARE BASIS)

	Fertilizer Treatment (kg/ha)												
	N	0	50	100	150	0	50	100	150	0	50	100	150
	P_2O_5	0	0	0	0	25	25	25	25	50	50	50	50
Net Yield (ton/ha)		1.99	2.83	3.52	3.61	2.20	3.49	3.96	4.36	2.12	3.64	4.27	4.64
Gross Field Benefits ($/ha)		1990	2830	3520	3610	2200	3490	3960	4360	2120	3640	4270	4640
Costs													
Nitrogen ($8/kgN)		0	400	800	1200	0	400	800	1200	0	400	800	1200
Phosphate ($10/kgP)		0	0	0	0	250	250	250	250	500	500	500	500
Labor for Application ($/ha)		0	50	100	100	50	50	100	100	50	50	100	100
Total costs that vary ($/ha)		0	450	900	1300	300	700	1150	1550	550	950	1400	1800
Costs with 40 percent capital Cost ($/ha)		0	630	1260	1820	420	980	1610	2170	770	1330	1960	2520
Net Benefits[a/]		1990	2200	2260	1790	1780	2510	2350	2190	1350	2310	2310	2120

[a/] Gross Field Benefits minus total cost that vary including capital cost.

Source: Modified from Perrin et al (1976): Table 3 to include capital costs.

TABLE 3.1. PARTIALLY BUDGET COMPARING TWO LEVELS OF NITROGEN AND PHOSPHOROUS TO ZERO FERTILIZER USE

	Treatment	
	50kg N/ha 0kg P/ha	40kg N/ha 25kg P/ha
Added Revenues		
Added Yield (ton/ha)	.93	1.67
Added Revenue ($/ha at $1000/ton)	930	1670
Added Costs		
Nitrogen ($8/kg N)	400	400
Phosphate ($10/kg N)	–	250
Cost of Application	50	50
Total Added Costs (excluding capital costs)	450	700
Cost of Capital (40 percent)	180	280
Total Added Costs	630	980
Added Profits	300	690

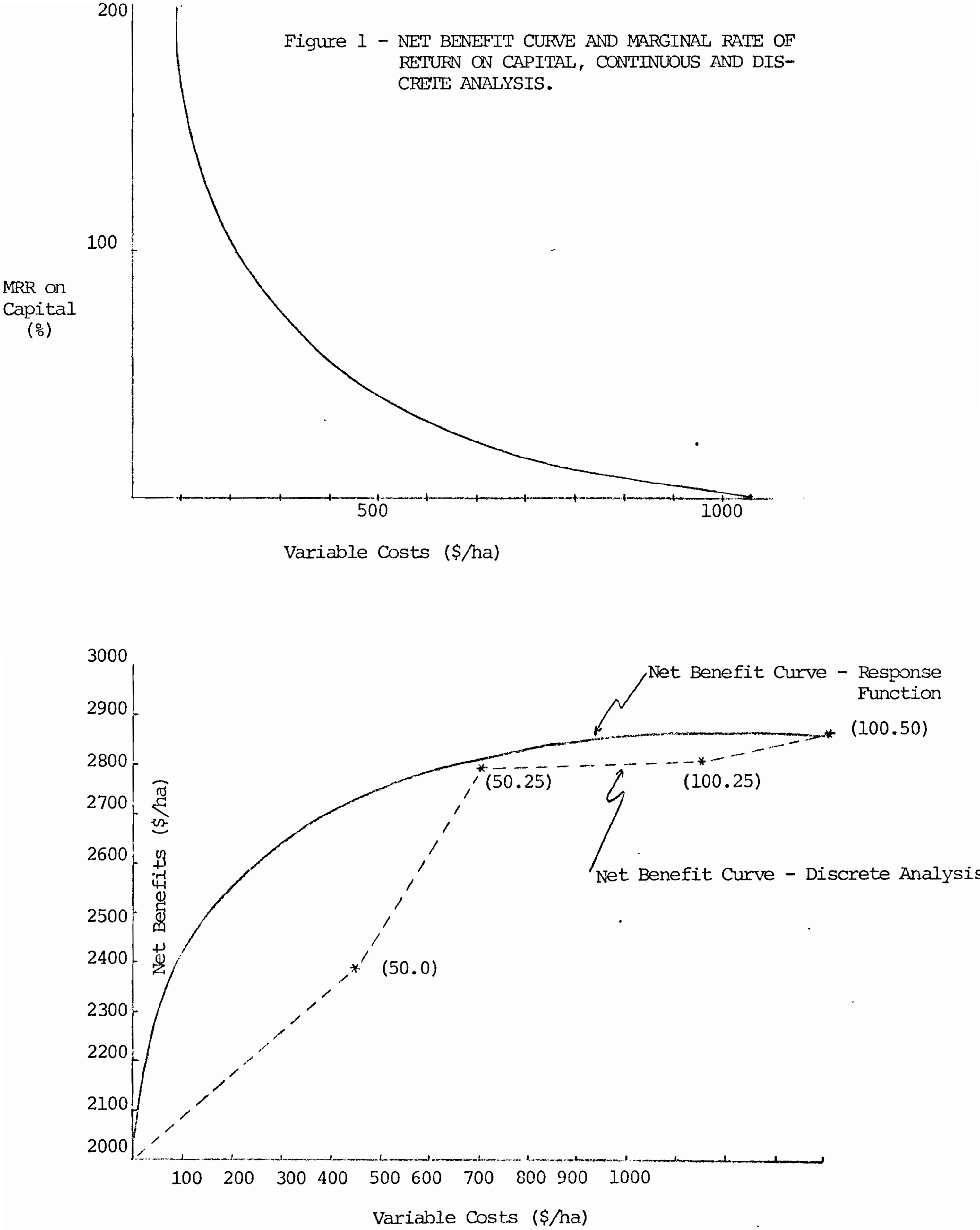


Figure 1 - NET BENEFIT CURVE AND MARGINAL RATE OF RETURN ON CAPITAL, CONTINUOUS AND DISCRETE ANALYSIS.

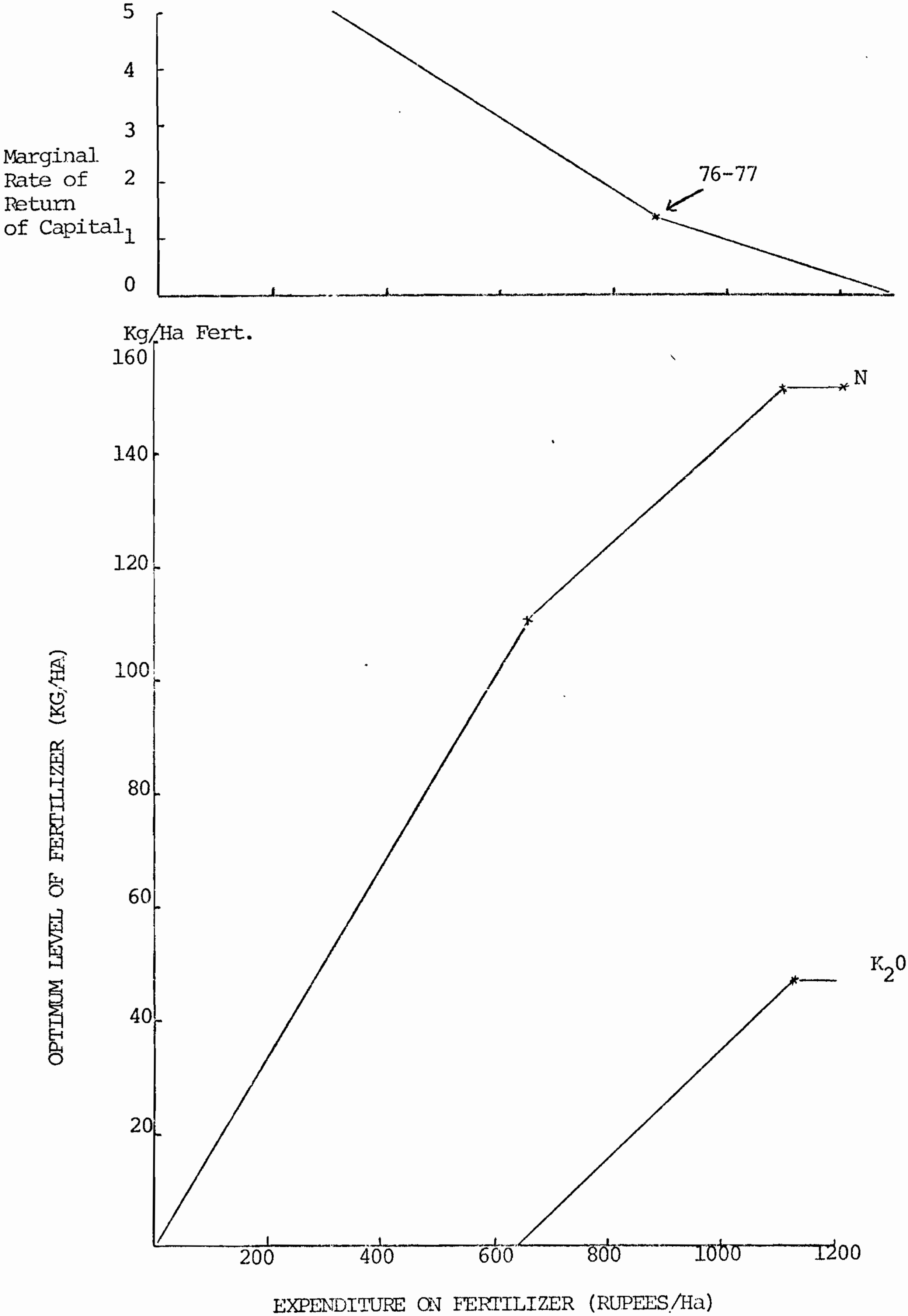
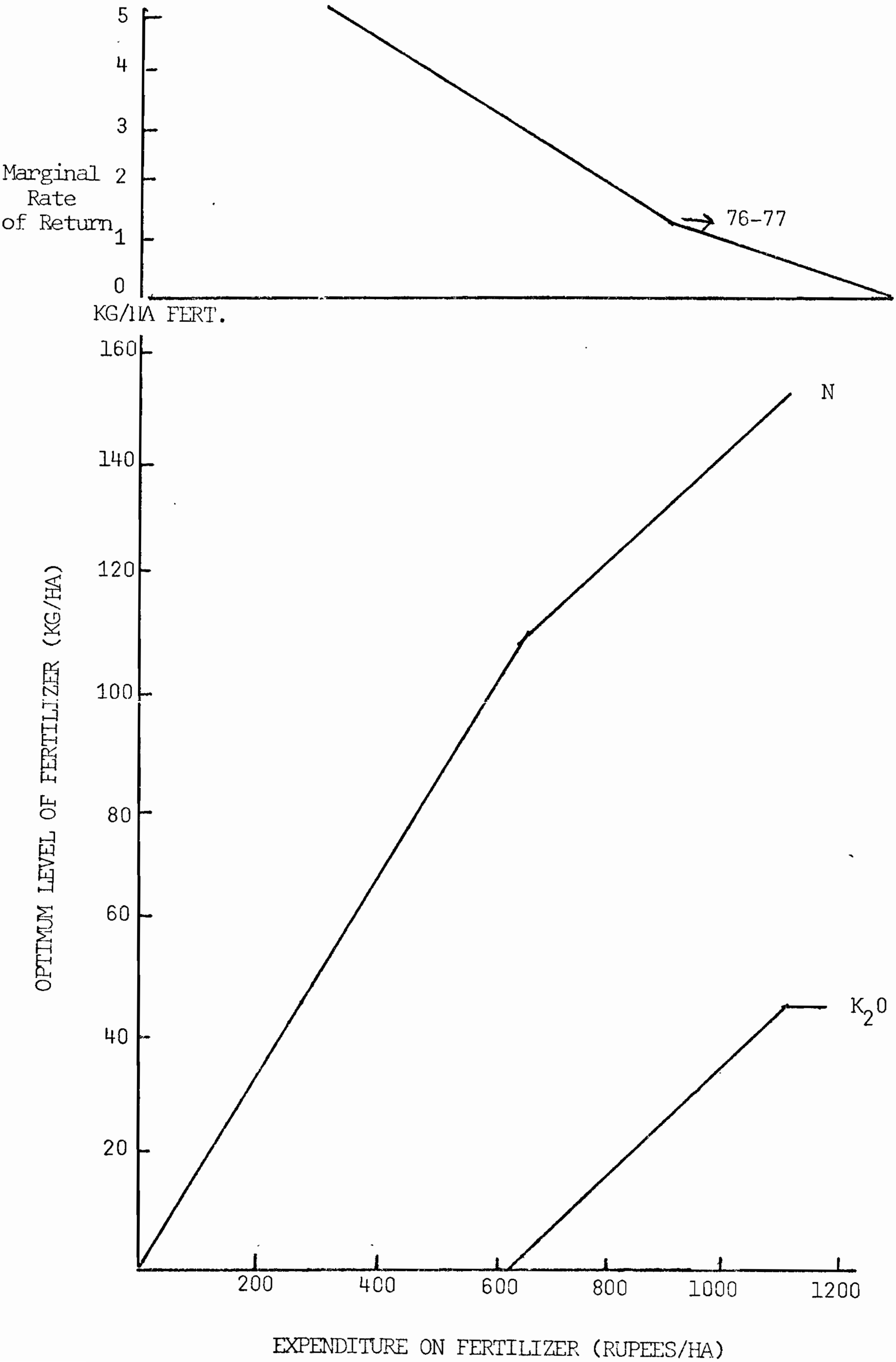


FIGURE 2: OPTIMAL RATE OF N AND K AND MRR ON CAPITAL INVESTED FOR IRRIGATED WHEAT, NEPAL

Source: Flinn (1979)



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