Measuring Stochastic Technology for the Multi-product Firm: The Irrigated Farms of Sudan

Arne Hallam[1], Rashid M. Hassan[2] and B. D'Silva[3]

[1]*Associate professor, Department of Economics, Iowa State University, Ames, Iowa.*
[2]*Research assistant, Department of Economics, Iowa State University, Ames, Iowa.*
[3]*Agricultural Economist, United States Department of Agriculture, Washington, D.C.*

Received 6 December, accepted 14 July 1989

Random production processes have important implications for allocative efficiency under risk aversion. The functional relationships between factor inputs and the first two moments of the distribution of crop yields in the Rahad scheme in Sudan are measured. Flexible functional specifications of the multi-product technology of the Rahad tenants are used to allow for risk-increasing as well as risk-decreasing effects of production inputs. Estimation procedures that correct for heteroscedasticity of the error structure, endogeneity of factor inputs and jointness in production are employed to estimate the parameters of the model. Standard errors of coefficients are reduced when heteroscedasticity is taken into account. The data reject misspecification due to endogeneity of factor inputs. The way hired labor is paid is found to determine its risk effect. Separability tests reject the aggregation of family and hired labor, whereas weeding and harvesting labor services are shown to be homogeneous. The observed structures have implications for employment patterns and relative wages if institutional policies should change to allow area allocations to respond to changing economic incentives.

Les procédés de production aléatoire ont un effet important sur l'allocation optimale des ressources en situation d'aversion pour le risque. On mesure, dans la présente étude, les rapports fonctionnels qui existent entre les facteurs de production et les deux premiers moments de la distribution des rendements des récoltes pour le projet Rahad, au Soudan. Les caractéristiques fonctionnelles souples de la technologie à production multiple des fermiers de Rahad permettent de tenier compte à la fois des effets de l'augmentation des risques et de la diminution des risques dus aux facteurs de production. Des méthodes d'estimation qui permettent de corriger les effets de l'hétéroscédasticité des termes d'écart, de l'endogénéité des facteurs de produciton et du caractère communautaire de la production servent à déterminer les paramètres du modèle. Les erreurs-type des coefficients sont réduites lorsque l'on tient compte de l'hétéroscédasticité. Les données portent à rejetter le défaut de sensibilité dû à l'endogénéité des facteurs de production. On constate par ailleurs que la méthode de rémunération de la main-d'oeuvre salariée est déterminante en ce qui concerne les effets de risque. Les tests de séparabilité portent à rejeter l'hypothèse de l'homogénéité des familles et de la main-d'oeuvre salariée mais confirment cette homogénéité dans le cas des équipes de sarclage et de récolte. Les structures observées sont importantes du point de vue des dispositions d'embauche et des salaires relatifs dans une situation où l'on songerait à modifier la répartition des superficies cultivés de manière à tenir compte de l'évolution de la situation économique.

INTRODUCTION

It is well recognized in the production economics literature that crop yields are random. The notion that factor inputs enter the higher moments (variance and skewness) of the distribution of output and thus influence production risks is empirically supported (Day 1965; Fuller 1965; Anderson 1973; de Janvry 1972; Just and Pope 1979; Antle and Goodger 1984). Under certainty or with uncertainty and expected profit maximization, producers equate the (expected) marginal value profit of inputs with factor cost. If producers maximize expected utility and use a safety-first decision mechanism or an alternative nonlinear decision rule regarding expected wealth, optimal input choices may depend on moments of the return distribution other than the mean. The ability to affect output distribution by input choice thus has implications for the supply behavior of risk-averse agricultural producers. Factor demands and adoption rates of various farming methods and production technologies by the risk-averse firms depend not only on yield effects but also on the risk effects of such decision variables. This fact makes the comprehensive characterization and measurement of the risk attributes of the production technology crucial to understanding farmers' behavior and to designing successful agricultural policies, regardless of the exact form of the preference mechanism.

Just and Pope (1978) criticize the traditional formulations of stochastic production functions for their implicit assumption of positive marginal risks for all inputs. They propose an alternative specification that allows for risk-decreasing as well as risk-increasing effects of factor inputs. Another alternative, the flexible moment-based (FMB) approach was introduced by Antle (1983) to estimate positive and negative risk effects of production factors. Both methods have been employed to measure stochastic technology in a single output framework (Just and Pope 1979; Griffiths and Anderson 1982; Antle and Goodger 1984). While the cited studies correct for the heteroscedastic structure of the technology, they do not deal with the possible endogeneity of factor inputs (but see Antle 1988, which does) or analyze multiple product firms. The present study uses survey data to measure and test the structure of the stochastic technology in the multi-crop farming system of the Rahad Scheme in Sudan using flexible functional forms. Estimation procedures that correct for heteroscedasticity, possible endogeneity of factor inputs and jointness in production are employed for efficient estimation of the technology parameters. Single equation methods as well as systems procedures are used to estimate crop yield moment functions.

The section below discusses the theoretical aspects of the specification of the stochastic technology. Joint production is described in the following section. Next the estimation procedures are defined. Then the empirical model and results are discussed, and the last section summarizes the findings.

PRODUCTION UNCERTAINTY AND STOCHASTIC TECHNOLOGY

One way to represent random technology is by a conditional probability distribution function rather than a single function of output (Day 1965; Anderson 1973; Roumasset 1976; Antle 1983):

$$F(y|x, \beta) \tag{1}$$

Production is considered a stochastic process for output, y, conditional on inputs, x, and the technology parameter vector β. Alternatively a random disturbance term can be appended to the deterministic neoclassical production function to represent stochastic technology (the production function representation):

$$y = f(x, \beta, \epsilon) \tag{2}$$

The latter representation is used in this study. One major concern in incorporating an error term is to investigate the implications of alternative specifications of the stochastic component, ϵ, on econometric estimation of the mean function (Marschak and Andrews 1944; Mundlak and Hoch 1965; Zellner, Kmenta and Dreeze 1966). The role of risk in decision making creates further interest in the effects of factor inputs on the behavior of the higher moments of the distribution.

It has been argued that part of the variability in crop yields is explained by controllable factors such as irrigation, fertilizers, improved seeds, cultivation methods, etc. (Day 1965; Fuller 1965; de Janvry 1972; Just and Pope 1978; Pope and Kramer 1979). Other factors not usually considered related to risk may also affect yield variability, especially if they are the inputs over which the producer has the most direct contact. If agricultural producers can influence production risks by varying the levels of input use, then factor demands under risk aversion are different from those under risk neutrality. A risk averter will use more (less) of the risk-reducing (increasing) factor than the risk-neutral firm (Pope and Kramer 1979).

Various specifications have been used to represent stochastic technology. The main deficiency of many common forms is the implication that all factors are risk increasing and thus are all used less under risk aversion (Just and Pope 1978; Ratti and Ullah 1976; Batra and Ullah 1974). According to several reasonable risk considerations suggested by Just and Pope (1978), common types of additive and multiplicative disturbance forms are found lacking. An alternative, more flexible form is proposed by Just and Pope (1978):

$$y = f(x, \beta) + h(x, \alpha)\epsilon \tag{3a}$$

or

$$y = f(x, \beta) + u \tag{3b}$$

where $u = h(x, \alpha)\epsilon$. The model in Eq. 3 allows for separate effects of factor inputs, x, on the deterministic, f, and the stochastic, h, components of production. This formulation also allows for both risk-increasing and risk-reducing effects, e.g., h' ´ 0. The general model in Eq. 3 is used in the present study. More general error structures involving third moments could also be used (Antle 1983) but are rejected, given the data available.

JOINT PRODUCTION AND THE MULTI-PRODUCT FIRM

While there are no technical interdependencies among outputs, jointness in production is assumed in the study case because of the presence of allocable fixed resources, i.e., labor (Pfouts 1961). Jointness is generally modeled in the primal space by the multi-output production function given by:

$$F(y, X, Z) = 0 \tag{4}$$

where

y = a (K × 1) vector of outputs,
x = a (K × J) matrix of variable input allocations, and
z = a (K × M) matrix of fixed input allocations,

such that the physical constraint on fixed input Z_i and the aggregate use X_j is:

$$(Z_i, X_j) = \sum_{k=1}^{K} (z_{ki}, x_{kj}) \qquad \text{for all } i, j \tag{5}$$

where

z_{ki} (x_{kj}) = allocation of fixed factor i (variable factor j) to the kth output and
K = the number of outputs.

The large number of parameters contained in Eq. 4 may present a problem with degrees of freedom, as many products, y_k, and input allocations z_{ki}, x_{kj}, are involved in the estimation. Another limitation of Eq. 4 is a problem of missing data, when only aggregate levels, Z_i, X_j, rather than input allocations, z_{ki}, x_{kj}, are recorded.

As discussed by Just et al (1983) when jointness is due to allocated inputs, with available data on inputs, the most straightforward estimation procedure is to postulate a nonjoint inputs production function that determines output, y_k, from inputs, z_{ki}, x_{kj}. The implicit function in Eq. 4 can be written as:

$$y_k = G_k (X_k, Z_k) \qquad k = 1,\ldots,K \tag{6}$$

where outputs are linked only by the constraint on Z in Eq. 5.

APPROPRIATE ESTIMATION METHODS

A multi-stage nonlinear generalized least squares method (MNLS) has been suggested to estimate the parameters β and α of Eq. 3 (Just and Pope 1978; Griffiths and Anderson 1982). This suggested procedure extends the error components approach of Hoch (1962), Wallace and Hussain (1968) and Fuller and Battese (1973) to nonlinear models with both firm and time disturbance components. The MNLS method is briefly outlined below.

For the model in Eq. 3, let $E[\epsilon] = 0$ and $V(\epsilon) = \sigma$. Therefore, $E[y] = f(x,\beta)$ and $V(y) = h^2(x, \alpha)\ \sigma$. The MNLS estimator of β and α is obtained with the following procedure:

Step a: A consistent estimator for β is obtained in the first stage by nonlinear least squares (NLS) from the regression of y on $f(x, \beta)$. A consistent estimator of u is $\hat{u}$ and is given by $\hat{u} = y - f(x, \hat{\beta})$.

Step b: A consistent NLS estimator of α is then obtained in the second stage from the regression of $\hat{u}^2$ on $h^2(x, \alpha)$ where $h^2(x, \alpha)$ is a nonlinear function of x and a parameter vector α. Note that $E(u^2) = h^2(x, \alpha)\sigma$. The consistent estimator of α is then used to derive $h(x, \hat{\alpha})$.

Step c: A NLS estimator of β is then obtained from the weighted regression of y^* on $f^*(x, \beta)$ where $(y^*, f^*) = h(x, \hat{\alpha})^{-1}[y, f(x, \beta)]$.

The estimator of β obtained in step c (MNLS) is consistent and asymptotically efficient under a broad range of conditions (Just and Pope 1978). The asymptotic efficiency of the MNLS procedure has also been shown to hold when the disturbance term, ϵ, includes both cross section as well as time series components in Just and Pope (1979) and Griffiths and Anderson (1982).

The above procedure corrects for the heteroscedastic structure of Eq. 3. It does not, however, provide for jointness nor does it handle the possible endogeneity of factor inputs in the case of survey data.

While the data generated by controlled experiments do not contain behavioral restrictions, survey data, on the other hand, represent optimal choices of the sampled firms. When working with survey data, observed input and output levels are jointly determined by the first-order equations of the optimizing firm. Depending on the objective function of decision makers and the structure of the econometric model, these additional conditions may lead to bias in least squares estimation procedures. Marschak and Andrews (1944) point out that production function disturbances are transmitted to the system of first-order condition equations of factor demands and output supply, leading to endogeneity of input levels when producers maximize profits. Therefore the application of the procedure described above could result in simultaneous equation bias in parameter estimates when survey data are employed. Zellner, Kmenta and Dreeze (1966) have shown that production disturbances unknown at the time of decision making are not transmitted to factor use equations when maximization of expected profits is assumed. Blair and Lusky (1975) extend the Zellner et al result for expected profit maximization to expected utility maximization.

On the other hand, it has been shown that estimation biases can occur when using least squares in the presence of technological uncertainty (Feldstein 1971) or under measurement errors (Blair 1974).

To allow for possible endogeneity in a model with variance effects, the MNLS procedure was modified to handle the simultaneity problem. The NLS estimator obtained in step a of the above procedure was replaced by an instrumental variable estimator. This is equivalent to the nonlinear two stage least squares (N2SLS) procedure. This estimator, called instrumental variable nonlinear least squares (IVNLS), yields consistent estimators of β, $f(x, \beta)$ and hence u in step a (Amemyia 1974) if endogeneity is present. In step b, the heteroscedastic structure $h(x, \alpha)$ is estimated using the consistent IVNLS estimator of u obtained in step a. The consistent estimator of $h(x, \alpha)$ is then used in the third-stage weighted regression of y^* on $f^*(x, \beta)$ to obtain the instrumental variable multi-stage nonlinear (IVMNLS) estimator of β.

Jointness in production of the crops modeled in this paper is implied by allocated fixed resources at the farmers' disposal, as discussed earlier. Allocative efficiency errors of the optimizing firm are therefore assumed to be interdependent in all the first-order optimality conditions equations. Accordingly, survey data representing the optimal choices of optimizing farmers like those in the study region are expected to exhibit cross-equation correlations between disturbances in the technology functions to be estimated. A systems procedure is more efficient than the single equation methods described above for such a system. For the case of no simultaneity, the systems version of the multi-stage least squares estimator (MNSUR) is used. A nonlinear simultaneous equations method that corrects for heteroscedasticity, endogeneity of factor inputs and cross equation correlations is employed in the case of simultaneity. We refer to this procedure as the instrumental variable multi-stage nonlinear seemingly unrelated regression estimator (IVMNSUR). The asymptotic properties of the nonlinear systems estimators are established in Barnett (1976), Amemyia (1977), Gallant (1975), Gallant and Jorgenson (1979) and Gallant (1987). The MNLS, and MNSUR, IVMNLS, and the IVMNSUR procedures are employed to estimate the technology parameters of the empirical model developed next. While the procedures used are consistent in large samples, they may be biased in small samples. Whether our sample size of 54 observations results in small sample bias is an unanswered empirical question.

THE FARMING SYSTEM

The study area is the Rahad, the second-largest irrigation scheme in Sudan. Cotton, groundnuts and sorghum are grown in the Rahad Scheme under regular irrigation and mechanical power over an area of 300,000 feddans (1 feddan = 0.411 hectares). The scheme is managed by the Rahad Agricultural Corporation, a parastatal organization. Farming is practiced under a scheme-mandated rotation with highly centralized decision making. A fixed size tenancy of 22 feddans is alloted to each farmer by the scheme administration. Half the land is planted to cotton, while groundnuts and sorghum share equally the other half of the tenancy in an intensive cropping sequence.

The corporation in this arrangement stands as the owner of the land and the irrigation network. In addition to renting out these resources to the tenant farmers, the corporation controls and directly performs the critical mechanical operations, provides most of the nonlabor inputs and some credit for cotton production. Land rent, together with charges for water, materials and services provided by the corporation, are deducted from proceeds of the farmers' cotton crop, which is delivered to the corporation for ginning and marketing. Sorghum, the main food staple, is usually retained for home consumption and groundnuts are sold by the individual tenant to private buyers.

The major agricultural operations such as plowing, ridging, planting, aerial spraying of pesticides, and application of fertilizers and herbicides are directly performed by the scheme personnel or contracted for with other companies. The number of irrigations and quantity of water discharged for each individual farmer is completely controlled by the scheme's irrigation authority. Equal rates of seeds and chemical inputs as recommended by the scheme's research station are applied to all tenancies. Chemical inputs, however, are applied only on cotton fields, the government crop. The corporation also provides cash advances to support the hiring of cotton labor. Cotton picking is closely supervised by the scheme's large team of field inspectors.

Under this administered system, labor is the only input for which the tenant has allocation flexibility both during the season and across the three crops grown. Thus, except for family and hired labor allocations, other inputs are fixed and identical for all farmers. Whereas most studies aggregate labor because of the confounding effects of other inputs, with controlled inputs this study provides an opportunity to effectively measure the mean and risk effects of different types of labor inputs.

Fixed wages are paid to sorghum and cotton laborers, whereas sharecropping arrangements dominate labor hiring in groundnuts production. While the present study does not address the question of choice of labor contracts as its major concern, the following reasons for this choice are observed. First, it is observed that farmers use most of their family labor resources in sorghum production as the most important crop, providing food security and subsistence to the farming family. Coupled with the fact that cotton operations are closely supervised by field inspectors, this implies easier or less expensive monitoring of labor in both the sorghum and cotton fields relative to groundnut fields. On the other hand, the overlap between the highly labor-intensive groundnut weeding and harvesting with sorghum and cotton operations makes it difficult to monitor and enforce labor hiring contracts on the groundnut fields, hence the existence of share cropping in this enterprise. These observations are in line with analytical and empirical results obtained elsewhere (Keijiro and Hayami 1988; Raumasset and Uy 1980; Masao and Hayami 1980; Alston 1981) on share tenancy arrangements for unenforceable contracts.

While area allocations and purchased inputs are fixed by the scheme administration, crop yields may be responsive to the quality and level of the labor and managerial resources under farmers' control. Crop yield functions are thus specified to depend on labor allocations, sowing dates and managerial ability, and farming skills of the tenant farmers. The experience of farmers in managing the various crop enterprises, measured as the number of years in farming each crop, is used as a proxy to managerial ability and skill. Weeding and harvesting are identified to be the major activities that employ labor. Family and hired labor are the sources of labor supply.

THE ECONOMETRIC MODEL

Three yield equations are specified to represent the multi-crop production technology of the Rahad tenants. Flexible functional forms are utilized in estimating the mean and variance of yield functions of Eq. 3 and testing for the technology structure. In the unrestricted form of the functions, there are six factors, specifically, sowing date, *SD*, years in farming crop k, *FR*, family and hired weeding labor, *WF*, *WH*, and family and hired harvesting labor, *HF*, *HH*. While sowing dates and years of farming are considered exogenous, weeding and harvesting family labor and weeding and harvesting hired labor are assumed to be endogenous. Sets of instrumental variables are constructed for each of the endogenous labor variables in each of the three yield equations. The set of instrumental variables include age, sex, farming years, education, distance between tenancies and homesteads, family size, average wage rates, sowing, weeding and harvesting date overlap indexes, labor recruitment methods and labor origin.

Whereas other studies typically use Cobb-Douglas or quadratic specifications, the transcendental logarithmic function (Christensen, Jorgenson and Lau 1973) is used to estimate components f and h of Eq. 3:

$$\ln y_k = \alpha_{0k} + \sum_{i=1}^{n} \alpha_{ik} \ln x_{ik} + \sum_j \sum_i \beta_{ijk} \ln x_{ik} \ln x_{jk} \tag{7}$$

Symmetry is imposed on the translog function, $\beta_{ij} = \beta_{ji}$. Other structural features are statistically tested. Following Berndt and Christensen (1973), tests are performed for homogeneity, functional separability of the mean functions, and monotonicity and convexity.

A multi-stage, stratified simple random sample of 54 farmers is surveyed in the Rahad Scheme (1984–85). As the vast majority of farmers are illiterate and no records are kept on the different crop budgets, the required factual information is gathered during frequent visits made to area farms to minimize memory bias. The collected information is utilized to estimate the parameters of the three yield functions specified in Eq. 6 above.

Results of the various structural tests are summarized in Table 1. The data do not reject the factor-wise separable Cobb-Douglas structure for the groundnuts technology. Thus under a Cobb-Douglas structure, weeding and harvesting represent distinct operations in groundnuts production that require different skills and labor services. The structure also implies that family and hired labor provide distinct services and earn different wages in the weeding and harvesting of groundnuts. Homogeneity is, of course, implied by the Cobb-Douglas structure.

On the other hand, homogeneity is rejected for cotton and sorghum. The results in Table 1 fail to support the hypothesis that family and hired labor provide homogeneous services in cotton and sorghum production (the first four separability tests). However, the data support the hypothesis that aggregate indexes for weeding and harvesting labor services exist for cotton and sorghum (last three tests). These results imply that, while the elasticity of substitution between family and hired labor depends on the levels (or relative prices) of other inputs, substitution between weeding and harvesting labor is independent of other input levels in cotton and sorghum production. They also indicate that wages paid to hired labor cannot be used as a proxy to the marginal value product of family labor, whereas harvesting and weeding labor can be paid the same wages in cotton and sorghum production.

As weeding and harvesting operations are performed at different times during the agricultural season, the above separability results have significant implications for seasonal and structural unemployment in the scheme. The fact that weeding and harvesting do not require specialized skills reduces the possibilities of structural and seasonal unemployment of labor resources in cotton and sorghum. The reverse is true, however, for groundnuts, as labor mobility between seasonal jobs requires higher adjustment costs (special training). Similarly an expansion in the scale of production or introduction of labor-saving (mechanical harvesting) or labor-using technologies has different impacts on employment patterns and wages for the different labor categories.

The homogeneity results have several policy implications as well, if the institutional rigidities are removed and if relative prices influence area allocations in the scheme. They predict that a change in the cropping pattern in response to a shock in relative prices will not affect relative wages and labor input ratios within the groundnut enterprise, whereas wages and intensity of hired labor use will adjust for the nonhomogeneous structures in other crops. Intercrop wage rates, however, will adjust in response to changing relative output prices.

In accordance with the results in Table 1, admissible structures are then reestimated, imposing the accepted restrictions. Monotonicity and convexity are satisfied at 10 sample point evaluations for all crops. The elasticities given in Table

Table 1. Results of the statistical tests

Tests		Cotton		Sorghum		Groundnuts	
		Linear restrictions	Nonlinear restrictions	Linear restrictions	Nonlinear restrictions	Linear restrictions	Nonlinear restrictions
Homogeneity[a]							
F^2		3.57*	–	5.46**	–	1.19	–
X^2		8.14*	–	10.91*	–	2.38	–
Separability tests							
Complete separability	F	18.55***	–	3.96*	–	2.66	–
	X^2	74.19	–	15.39*	–	12.64	–
Separability in harvesting labor	F	34.91***	6.96**	4.23*	3.46*	2.42	1.55
between family and hired	X^2	69.81*	20.89**	13.21*	9.97*	4.83	4.65
Separability in weeding labor	F	18.76***	5.86*	5.13**	3.78*	2.26	0.78
between family and hired	X^2	37.51***	17.58**	10.17*	11.67	4.53	2.32
Separability in weeding and	F	17.74***	4.87**	4.71*	3.89*	1.94	0.937
harvesting labor	X^2	35.47***	24.02**	12.31*	14.43*	4.15	4.68
Separability in hired labor	F	2.19	3.84	2.91	1.98	1.66	0.94
	X^2	10.27	11.51	8.56	5.94	3.94	2.81
Separability in family labor	F	2.34	2.21	2.86	2.64	1.98	0.97
	X^2	11.11	6.61	7.14	6.92	3.47	2.89
Separability in hired and family	F	2.49	2.62	2.21	1.83	1.79	0.68
labor	X^2	12.48	9.26	7.43	6.17	2.45	3.42

[a]The F-values represent the likelihood ratio tested against $F(q, N - K)$, and the X^2 values are the Wald X^2 tested against $X^2_\alpha(q)$.
*significant at 10%
**significant at 5%
***significant at 1%

5 further support the hypothesis of variable elasticities of substitution and non-homogeneous structures for cotton and sorghum.

The estimated parameters of the restricted mean yield and variance functions are given in Tables 2 to 4. A Hausman (1978) test with a statistic of 0.173 is calculated to test for misspecification due to endogeneity of factor inputs on the system (MNSUR) and the three-stage (IVMNSUR) procedures. The result implies nonrejection of exogeneity for any tabled level of significance and for all values of degrees of freedom. This result tends to support expected utility or profit maximization and to refute the assumption regarding endogeneity of factor inputs. On the other hand, Tables 2 to 4 indicate the importance of the heteroscedastic structure of the production technology. A large improvement in parameter significance is realized when heteroscedasticity is taken into consideration (NLS versus MNLS and IVNLS versus IVMNLS). Jointness (cross-equation correlations), however, makes little difference and seems to be unimportant for estimation of the multi-product technology when input allocations are observed (MNLS versus MNSUR and IVMNLS versus IVMNSUR).

Parameter estimates of the second moment of the yield functions are given in Tables 2 to 4. The risk effects of factor inputs are evaluated and are reported below at the mean of the data. The mean values of all data are given in Table 6. Optimal sowing, *SD*, and farming experience, *FR*, are found to reduce production risks. Family labor is risk decreasing whereas hired labor is risk increasing for cotton and sorghum. The reverse is true for groundnuts. This suggests an interesting relationship between the form of labor contracts, e.g., the way hired labor is paid and its risk effects. While sharecropping arrangements dominate groundnuts production, where hired labor is risk reducing, cotton and sorghum hired labor are paid cash wages. In addition to other reasons suggested earlier for the existence of share contracts in groundnuts production, the negative risk effects of share tenancy arrangements may be considered one more explanatory factor. Family labor, on the other hand, is more skilled in cotton and sorghum operations, as it specializes in their production and leaves the groundnuts tenancy for sharecropping labor.

SUMMARY AND CONCLUSIONS

The uncertain nature of farming and the important role of risk in supply decisions necessitate the comprehensive characterization and measurement of the structure and risk attributes of the production technology. In the present study, the stochastic technology for the multi-product farmers of the Rahad irrigation scheme of Sudan is measured and its structure statistically tested. Stochastic production functions are specified for the three crops grown. Risk-increasing as well as risk-reducing input effects are allowed in the stochastic representation. Single equation methods and systems procedures are employed to estimate the parameters of the first two moments of the distribution of crop yields. Significant improvements are realized

Table 2. The single equation and systems methods estimates of the restricted translog function for cotton[a]

	First moment						Second moment
	NLS	MNLS	MNSUR	IVNLS	IVMNLS	IVMNSUR	
Constant	−5.21	−4.56	2.11	−1.15	−1.71	−1.83	3.42
	(−3.7)**	(−6.01)**	(2.4)*	(−2.99)*	(−3.9)**	(−2.7)**	(0.74)
SD[b]	0.31	0.20	0.35	0.386	0.45	0.40	−2.85
	(1.49)	(1.42)	(1.66)	(1.78)	(2.38)*	(2.6)**	(−2.25)*
SD^2	−0.04	−0.031	−0.04	−0.018	−0.02	−0.06	0.166
	(−0.62)	(−1.56)	(−4.1)**	(−3.04)**	(−2.9)**	(−3.7)**	(0.61)
FR	0.141	0.178	0.24	0.215	0.27	0.18	−0.57
	(0.46)	(2.64)**	(1.86)	(2.51)**	(4.65)**	(2.76)**	(−1.45)
FR^2	−0.02	−0.01	−0.03	−0.011	−0.06	−0.014	0.06
	(−1.43)	(−1.93)*	(−1.48)	(−1.93)	(−4.44)**	(−2.74)**	(1.38)
FL	0.296	0.375	0.52	0.341	0.435	0.418	−0.633
	(2.61)**	(3.59)**	(2.62)**	(2.24)*	(4.56)**	(3.28)**	(−1.76)
FL^2	−0.01	−0.006	−0.02	−0.024	−0.03	−0.023	−1.26
	(−1.4)	(−2.69)**	(−4.41)**	(−5.26)**	(−5.23)**	(−5.37)**	(−1.55)
HL	0.351	0.087	0.20	0.281	0.347	0.332	1.27
	(1.92)	(2.28)*	(2.34)*	(2.33)*	(2.19)*	(2.91)**	(2.38)*
HL^2	−0.12	−0.051	−0.13	−0.098	−0.01	−0.075	−0.0324
	(−0.94)	(−3.34)**	(−2.41)**	(−1.5)	(−2.12)*	(−2.18)*	(−1.33)
$FL{\cdot}HL$	0.224	0.015	0.22	0.185	0.05	0.019	−0.137
	(1.23)	(1.44)	(1.62)	(1.76)	(4.11)**	(1.87)	(−1.9)
R^2	0.86	0.87	0.90	0.93	0.89	0.93	0.43
D-W	2.14	2.53	2.21	2.01	1.76	–	–

[a]Single equation methods are the nonlinear (NLS), the multi-stage nonlinear (MNLS), the instrumental variable nonlinear (IVNLS) and the instrumental variable multi-stage nonlinear (IVMNLS) least squares estimators. Systems procedures, the multi-stage nonlinear (MNSUR) and the instrumental variable multi-stage nonlinear (IVMNSUR) are seemingly unrelated regression (Zellner) estimators.

[b]*SD* = sowing date, *FR* = years in farming, *HL* = hired labor, *FL* = family labor

Table 3. The single equation and systems methods estimates of the restricted translog function for sorghum[a]

	First moment						Second moment
	NLS	MNLS	MNSUR	IVNLS	IVMNLS	IVMNSUR	
Constant	−3.61	−1.49	−2.61	−0.91	1.96	0.86	16.09
	(−1.24)	(−2.14)*	(−2.80)**	(−3.51)**	(3.61)**	(2.98)**	(0.578)
SD[b]	0.12	0.09	0.24	0.12	0.137	0.104	−3.47
	(0.66)	(1.47)	(1.51)	(0.93)	(1.86)	(1.97)	(−2.21)*
SD^2	−0.05	−0.044	−0.11	−0.02	−0.03	−0.02	1.04
	(−1.31)	(−1.12)	(−1.44)	(−0.99)	(−2.3)*	(−2.01)*	(1.11)
FR	0.34	0.092	0.27	0.11	0.26	0.14	−1.15
	(2.74)**	(2.91)**	(2.99)**	(3.11)**	(4.72)**	(3.66)**	(−1.41)
FR^2	−0.03	−0.67	−0.12	−0.01	−0.022	−0.013	0.163
	(−1.27)	(−1.77)*	(−2.32)*	(−2.52)**	(−4.0)**	(−3.09)**	(1.67)
FL	0.08	0.243	0.31	0.186	0.217	0.28	−0.52
	(1.74)	(2.11)*	(1.94)	(2.29)*	(2.3)*	(2.11)*	(−1.4)
FL^2	−0.02	−0.08	−0.11	−0.05	−0.08	−0.048	0.003
	(−2.06)*	(−5.21)**	(−3.21)**	(−6.77)**	(−4.94)**	(−6.63)**	(0.46)
HL	0.108	0.216	0.24	0.17	0.18	0.21	1.03
	(1.69)	(1.62)	(1.83)	(2.01)*	(2.78)**	(2.85)**	(1.93)*
HL^2	−0.09	−0.134	−0.14	−0.046	−0.08	−0.05	−0.49
	(−1.82)	(−2.02)*	(−2.32)*	(−3.62)**	(−4.1)**	(−3.4)**	(−0.42)
$FL \cdot HL$	0.002	0.033	0.023	0.019	0.016	0.02	−0.05
	(1.63)	(2.94)**	(1.64)	(1.42)	(4.94)**	(2.77)**	(−0.03)
R^2	0.76	0.79	0.72	0.83	0.78	0.84	0.39
$D\text{-}W$	1.85	1.88	2.14	1.95	1.91	–	–

[a]Single equation methods are the nonlinear (NLS), the multi-stage nonlinear (MNLS), the instrumental variable nonlinear (IVNLS) and the instrumental variable multi-stage nonlinear (IVMNLS) least squares estimators. Systems procedures, the multi-stage nonlinear (MNSUR) and the instrumental variable multi-stage nonlinear (IVMNSUR) are seemingly unrelated regression (Zellner) estimators.
[b]SD = sowing date, FR = years in farming, HL = hired labor, FL = family labor

Table 4. The single equation and systems methods estimates of the restricted translog function for groundnuts (Cobb-Douglas)[a]

	First moment						Second moment
	NLS	MNLS	MNSUR	IVNLS	IVMNLS	IVMNSUR	
Constant	−1.97	−2.87	1.13	−2.45	0.68	0.82	7.42
	(−4.21)**	(−2.11)*	(5.44)**	(−3.97)**	(9.82)**	(9.89)**	(1.62)
SD[b]	0.317	0.04	0.20	0.22	0.167	0.14	−0.41
	(0.606)	(0.17)	(1.46)	(1.67)	(3.0)**	(2.8)**	(−2.3)*
FR	0.14	0.154	0.19	0.12	0.056	0.034	−0.82
	(0.85)	(2.81)**	(1.87)	(2.1)*	(3.7)**	(12.35)**	(−0.89)
WF	0.09	0.107	0.04	0.14	0.164	0.112	0.21
	(1.2)	(1.49)	(2.08)*	(1.21)	(1.94)*	(2.98)**	(1.24)
WH	0.25	0.29	0.11	0.19	0.13	0.214	−0.28
	(1.01)	(1.91)	(1.32)	(0.98)	(1.11)	(1.84)*	(−1.92)*
HF	0.11	0.14	0.21	0.17	0.067	0.121	0.03
	(1.16)	(2.04)*	(1.81)	(1.12)	(1.34)	(1.95)	(2.17)*
HH	0.09	0.27	0.25	0.16	0.415	0.38	−0.07
	(2.11)**	(4.36)**	(3.4)**	(2.1)*	(4.32)**	(6.54)**	(−1.73)
R^2	0.62	0.68	0.77	0.71	0.76	0.74	0.62
D-W	1.94	1.89	1.94	2.14	1.81	2.1	–

[a]Single equation methods are the nonlinear (NLS), the multi-stage nonlinear (MNLS), the instrumental variable nonlinear (IVNLS) and the instrumental variable multi-stage nonlinear (IVMNLS) least squares estimators. Systems procedures, the multi-stage nonlinear (MNSUR) and the instrumental variable multi-stage nonlinear (IVMNSUR) are seemingly unrelated regression (Zellner) estimators.

[b]*SD* = sowing date, *FR* = years in farming, *HL* = hired labor, *FL* = family labor

Table 5. Scale (ϵ) and labor substitution elasticities (σ_{ij}) for the cotton and sorghum crops at 10 sample points

	Cotton		Sorghum	
	ϵ	σ_{ij}[a]	ϵ	σ_{ij}[a]
1	0.9185	0.5352	0.6973	0.0173
2	1.3117	0.4205	0.6761	0.1991
3	1.0142	0.6124	0.6778	−0.3414
4	0.9901	0.356	0.7226	0.9936
5	0.8906	0.6003	0.4291	1.0757
6	1.0997	0.8211	0.3494	0.4242
7	1.0075	0.7443	0.7898	0.6352
8	1.1766	0.7007	0.7009	2.2149
9	0.7735	0.0232	0.7691	0.5641
10	1.0648	0.4145	0.7511	3.6432

[a]σ_{ij} denotes the elasticity of substitution between family and hired labor. Note that scale and substitution elasticities for groundnuts are known by the assumption of a Cobb-Douglas structure.

in parameter estimation when the heteroscedastic structure of production disturbances is taken into consideration. Endogeneity of factor inputs and jointness in production does not prove important in estimating the technology parameters of the Rahad farmers.

Separability tests show that family and hired labor perform distinct tasks and do not support their aggregation. The existence of aggregate indexes for weeding and harvesting labor is supported. The way hired labor is paid influences its risk effects. While hired labor increases production risk in cotton and sorghum, where cash wages are paid, it is risk decreasing in groundnuts production, where sharecropping prevails. Family labor is found to be risk reducing in cotton and sorghum production. The observed structures have significant implications for employment patterns and relative wages if institutional policies should change to allow area allocations to respond to changing economic incentives.

ACKNOWLEDGMENT

The authors are grateful to Rulon Pope for helpful comments and suggestions.

REFERENCES

Alston, Lee J. 1981. Tenure choice in southern agriculture, 1930–1960. *Explorations in Economic History* 18 (July): 211–32.

Amemyia, T. 1974. The non-linear two stage least squares estimator. *Journal of Econometrics* 2: 105–10.

Table 6. Sample statistics summary

	Cotton		Groundnuts		Sorghum	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
Yield (kantar per feddan)	5.6	8.2	28	9.7	12	5.8
Sowing date (index)[a]	2.2	1.4	2.8	1.8	2.1	2.2
Years in farming (*FR*)	6	0.96	12	4.9	24	18.2
			(person-days per feddan)			
Family labor (*FL*)	6.2	7.4	5.2	12.3	7.3	4.3
Hired labor (*HL*)	32.5	11.9	27.7	15.6	21.4	9.4
Weeding family			2.45	22.7		
Weeding hired			11.89	19.6		
Harvesting family			1.82	16.4		
Harvesting hired			9.94	18.8		

[a]Sowing date index measures the deviation from an optimal sowing date in weeks. The optimal date is picked as the one that gives the highest average yield.

Amemyia, T. 1977. The maximum likelihood and non-linear three stage least squares estimator in the general non-linear simultaneous equations model. *Econometrica* 45: 955–68.

Anderson, J. R. 1973. Sparse data, climatic variability, and yield uncertainty in response analysis. *American Journal of Agricultural Economics* 55: 77–82.

Antle, J. 1983. Testing the stochastic structure of production: A flexible moment based approach. *Journal of Business and Economic Statistics* 1: 192–201

Antle, J. 1988. *Pesticide Policy, Production Risk, and Producer Welfare: An Econometric Approach to Applied Welfare Economics*. Washington, D.C.: Resources for the Future.

Antle, J. and Goodger. 1984. Measuring stochastic technology: The case of tulare milk production. *American Journal of Agricultural Economics* 66: 342–50.

Barnett, W. A. 1976. Maximum likelihood and the iterative Aitken estimation of non-linear systems of equations. *Journal of the American Statistical Association* 71: 354–60.

Batra, R. N. and A. Ullah. 1974. Competitive firm and the theory of input demand under price uncertainty. *Journal of Political Economy* 82: 537–48.

Berndt, E. R. and L. P. Christensen. 1973. The translog and the substitution of equipment, structures and labor in U.S. manufacturing 1929–68. *Journal of Economics* 1: 81–114.

Blair, R. D. 1974. Estimation of the elasticity of substitution when input prices are random. *Southern Economic Journal* 41.

Blair, R.D. and R. Lusky. 1975. A note on the influence of uncertainty on estimation of production function models. *Journal of Econometrics* 3: 391–94.

Christensen, L. R., D. W. Jorgenson and L. J. Lau. 1973. Transcendental logarithmic production frontiers. *Review of Economic Statistics* 55: 28–45.

Christensen, L. R., D. W. Jurgenson and L. J. Lau. 1971. Conjugate duality and the transcendental logarithmic production function. *Econometrica* 39.

Day, R. H. 1965. Probability distribution of field crops. *Journal of Farm Economics* 47.

de Janvry, A. 1972. Optimal levels of fertilization under risk: The potential for corn and wheat fertilization under alternative price policies in Argentina. *American Journal of Agricultural Economics* 54: 1–10.

Diewert, W. E. 1971. An application of the Shepherd duality theorem: A generalized Leontief production function. *Journal of Political Economy* 79: 481–507

Feldstein, M. S. 1971. Production with uncertain technology: Some economic and econometric implications. *International Economic Review* 12 (1): 27–38.

Fuller, W. A. 1965. Stochastic fertilizer production functions for continuous corn. *Journal of Farm Economics* 47: 105–19.

Fuller, W. A. and G. E. Battese. 1973. Transformations for estimation of linear models with nested error structure. *Journal of the American Statistical Association* 68: 626–36.

Gallant, A. R. 1975. Three stage least squares estimation for a system of simultaneous, non-linear, implicit equations. *Journal of Econometrics* 5: 71–88.

Gallant, A. R. 1987. *Non-linear Statistical Models*. New York. John Wiley and Sons.

Gallant, A. R. and D. W. Jorgenson. 1979. Statistical inference for a system of simultaneous, non-linear, implicit equations. *Journal of Econometrics* 5: 71–88.

Griffiths, W. E. and J. R. Anderson. 1982. Using time series and cross section data to estimate a function with positive and negative marginal risks. *Journal of the American Statistical Association* 77: 529–36.

Hall, R. E. 1973. The specification of technology with several kinds of output. *Journal of Political Economy* 81: 878–92.

Hausman, J. A. 1978. Specification tests in econometrics. *Econometrica* 46 (6): 1251–71.

Hoch, I. 1962. Estimation of production function parameters combining time-series and cross-section data. *Econometrica* 30: 34–54.

Just, R. E. and R. Pope. 1978. Stochastic specification of production functions and econometric implications. *Journal of Econometrics* 7: 67–86.

Just, R. E. and R. Pope. 1979. Production function estimation and related risk considerations. *American Journal of Agricultural Economics* 61: 276–84.

Just, R., D. Zilberman and E. Hochman. 1983. Estimation of multicrop production functions. *American Journal of Agricultural Economics* 65: 770–80.

Keijiro, Otsuka and Yujiro Hayami. 1988. Theories of share tenancy: A critical survey. *Economic Development and Cultural Change* 37 (October): 31–68.

Klein, L. R. 1947. The use of cross section data in econometrics with application to a study of production of railroad services in the United States. Washington, D.C.: National Bureau of Economic Research.

Marschak, J. and W. H. Andrews. 1944. Random simultaneous equations and the theory of production. *Econometrica* 12: 143–205.

Masao, Kikuchi and Yujiro Hayami. 1980. Technology and labor contract: Two systems of rice harvesting in the Philippines. *Journal of Comparative Economics* 4 (December): 357–77.

Mundlak, Y. and I. Hock. 1965. Consequences of alternative specifications in estimation of Cobb-Douglas production functions. *Econometrica* 33 (October): 814–28.

Pfouts, R. W. 1961. The theory of cost and production in the multi-product firm. *Econometrica* 29: 650–58.

Pope, R. and R. Kramer. 1979. Production uncertainty and factor demands for the competitive firm. *Southern Economic Journal* 46: 489–501.

Powell, A. and F. H. Gruen. 1968. The constant elasticity of transformation production function and linear supply system. *International Economic Review* 9: 315–28.

Ratti, R. and A. Ullah. 1976. Uncertainty in production and the competitive firm. *Southern Economic Journal* 43: 703–10.

Roumasset, J. A. 1976. *Rice and Risk: Decision Making among the Low-income Farmers*. Amsterdam: North Holland.

Roumasset, J. and M. Uy. 1980. Price rates, time rates, and teams: Explaining patterns in the employment relation. *Journal of Economic Behavior and Organization* 1 (December): 343–60.

Wallace, T. D. and A. Hussain. 1969. The use of error components models in combining cross-section with time series data. *Econometrica* 37: 55–72.

Zellner, A., J. Kmenta and J. Dreeze. 1966. Specification and estimation of Cobb-Douglas production function models. *Econometrica* 34: 784–95.