

Deriving Fertilizer Recommendations with a Flexible Functional Form

Gustavo E. Sain* and Mario A. Jauregui

ABSTRACT

General fertilizer response models are useful to derive robust recommendations for farmers who face different circumstances. This paper shows that a flexible functional form such as the transcendental can be applied to develop both agronomically and economically sound recommendation tables using a set of soil test, climate, and management variables. Data from thirty-eight experiments on wheat response to N and P in the Humid Pampa, Argentina, were used to build a general response model that incorporates the initial levels of soil test measures (NO₃-N and Bray-1 P), rainfall from sowing to maturity, and previous crop. The economic analysis was performed for relevant nutrient/grain price ratios $r_n = 8.0$ and $r_p = 18.3$ for N and P, respectively. A table of fertilizer recommendations for wheat was derived for different combinations of rainfall expectations, previous crop, and soil test values. The following results were obtained: (i) the estimated model not only accounted for a significant percentage of the total variability in the dependent variable (lnY), with $R^2 = 0.56$, $P = 0.01$, but it also gave coefficients with signs in conformity with agronomic expectations; (ii) the economic optima for N (N^*) and P (P^*) are compatible with the range of optima computed with per-site economic analyses; (iii) N^* and P^* increased as rainfall increased; (iv) N^* was lower for wheat after soybeans than for wheat after maize; (v) N^* was higher for smaller values of soil N, while P^* was practically unaffected by soil N; and (vi) P^* was higher for smaller values of soil P, while N^* was affected by soil P, with a pattern that depended on both previous crop and rainfall. Similar tables can be derived for groups of farmers under different economic circumstances (i.e., different input/output price ratios).

IN MOST NATIONAL AGRICULTURAL RESEARCH SYSTEMS of developing countries, the present stage of the economic assessment of crop response to fertilization in on-farm research is discontinuous or discrete analysis (for a detailed treatment of this method, see CIMMYT, 1988). Increased access to microcomputer technology, however, has opened the possibility of moving toward continuous analysis of experimental results, using response functions.

For a number of reasons, continuous analysis of on-farm research experiments has advantages over discrete analysis. Most importantly, it facilitates the construction of general models in which data are pooled across sites and years, accounting for variability of factors associated with soil, weather, and agricultural practices. Also, continuous analysis can help the researcher to understand and interpret experimental data and can provide insight into both agronomic and economic issues. Furthermore, the possibility of running powerful statistical packages with microcomputers allows application of flexible functional forms to estimate the response function. A flexible functional form is defined as one that captures all the information conveyed by the set of data, except for stochastic aberrations of local maxima, minima, and inflexion points not warranted by agronomic knowledge.

Current biological and agronomic knowledge cannot support the use of a specific functional form to represent the true response model. In this sense, the complex reality between the addition of fertilizer to the soil and the gathering of yield data can be regarded as a black box in which some known and many unknown processes are integrated. As Sanchez et al. (1981) mentioned, "the issue of response model selection has been long and often vigorously debated . . ." and ". . . the number of models that have been proposed is staggering." Nelson et al. (1985) concluded that no single model can be recommended for all situations. Furthermore, regardless of the selection procedure used, the researcher can only hope that the best model has some agronomic rationale and produces estimates of economic optima that are free of bias.

There are substantial differences in the relative importance that each functional form gives to both the information conveyed by the data and the model's maintained hypothesis. For example, a model such as Mitscherlich's (Colwell et al., 1988) will provide a concave fit irrespective of whether the data show a concave, linear, or convex distribution pattern. This is a case in which the data have little weight on the resulting response function. On the other extreme, the discrete economic analysis is model-free and puts full weight on the data. Both the quadratic and square root polynomials put some weight on the data and some on the model's structure.

The objective of this paper is to show that a flexible functional form such as the transcendental function can be applied to develop both agronomically and economically sound recommendation tables using a model that combines experimental variables with a set of soil test, climate, and management variables. The transcendental function is a member of the generalized power function (de Janvry, 1972). It displays a high degree of flexibility especially if, as suggested by Debertin (1986), an interaction term is included when more than one nutrient is considered.

THE GENERAL RESPONSE MODEL

The general response function can be written as

$$Y = f(M_i, X_j, S_k, R_l) \quad [1]$$

where Y represents yield; M_i is a vector of i ($i = 1, \dots, I$) nutrients available to the plant in the soil at either limiting or nonlimiting levels; X_j represents a vector of j ($j = 1, \dots, J$) inputs that are under discretionary control of the farmer, such as variety, crop rotation, and week control; S_k is a vector of k ($k = 1, \dots, K$) variables that are known or can be measured at the beginning of the production cycle, but which are not under control of the farmer, such as soil type; and R_l is a vector of l ($l = 1, \dots, L$) random climate or pest incidence variables that are not known with certainty at the time of decision making and cannot be controlled by the farmer. Rainfall is a frequent example for R_l .

This classification should not be considered as a rigid one. A fixed X_j variable such as previous crop cannot

G.E. Sain, Economics Program, Int. Maize and Wheat Improvement Ctr. (CIMMYT), Lisboa 27, Col. Juárez, Deleg. Cuauhtémoc, Apdo. Postal 6-641, 06600 México, DF, Mexico; and M.A. Jauregui, Fac. de Ciencias Agrarias, Univ. Nac. de Cuyo, Almirante Brown 500, 5505-Chacras de Coria, Mendoza, Argentina. Received 22 Oct. 1990. *Corresponding author.

be modified at the time the farmer makes a fertilizer decision. In this respect, this variable is regarded as similar to the S_k variables, which are known but not under the farmer's control. Furthermore, rainfall is random and unknown at the time of decision making; however, different recommendations can be derived based on rainfall expectations for various areas within the same region.

Nutrient availability in the soil at sowing time is an important site variable affecting crop response to fertilization. As for the case of rainfall or any other relevant variable, omission of an appropriate soil fertility index produces biased coefficients in the estimated equation (Colwell et al., 1988). Hence, soil test variables should be used at the experimental stage in order to derive a general model with unbiased estimates of the coefficients. This is true even where it is not practical to obtain the necessary variables (e.g., in developing countries that lack reliable technical facilities for routine soil testing). For example, after reviewing some difficulties that may be encountered in the estimation of fertilizer response functions, Anderson (1968) concluded that soil testing should always accompany fertilizer response work to improve the precision of farmers' recommendations.

Two review articles dealing with the inclusion of soil fertility variables in response analysis have been published recently by Nelson et al. (1985) and Nelson (1987). According to these authors, there are two major approaches to include the necessary soil test information. One is the covariance approach, in which the soil nutrient in question is treated as any other independent variable in a polynomial equation. Thus, yield is not forced to be zero for $M_f = t_m = 0$, where M_f represents the applied fertilizer nutrient and t_m represents the soil test value for nutrient M .

The other approach, called the total nutrient approach, is based on the model proposed more than three decades ago by Hildreth (1957). This model was later used by Anderson and Nelson (1971) and Mombiela et al. (1981), among others. In this case, fertilizer nutrient (M_f) and soil nutrient (M_s) are combined into a single measure of total nutrient M as:

$$M = M_f + M_s \quad [2]$$

Table 1. Description, sample mean, and standard deviation of the variables included in the model.

Variable	Description	Mean	SD
N_f	Added N. Levels used were 0, 30, 60, 90, and 120 kg ha ⁻¹		
P_f	Added P. Levels used were 0, 8.7, 17.5, 26.2, and 34.9 kg ha ⁻¹		
N	Estimated as $N_f + d_n$ (kg ha ⁻¹)		
P	Estimated as $P_f + d_p$ (kg ha ⁻¹)		
t_n	N soil test (mg NO ₃ in 0-20 cm soil depth)	50 mg kg ⁻¹	25 mg kg ⁻¹
t_p	P soil test (mg Bray-1 P kg ⁻¹ in 0-20 cm soil depth)	13 mg kg ⁻¹	7 mg kg ⁻¹
R	Rainfall from sowing to maturity (mm)	249 mm	72 mm
C	Previous crop ($C = 0$ if maize and $C = 1$ if soybean)		

where the unknown M_s value is expressed in the same units of M_f (i.e., kg ha⁻¹). For computational purposes, M_s is replaced by a linear function of the soil test value, t_m :

$$M_s = a t_m \quad [3]$$

Substituting this expression into Eq. [2] yields:

$$M = M_f + a t_m \quad [4]$$

which is the equation used to calculate M from the values of M_f and t_m (Mombiela et al., 1981). In this approach, the function should be estimated forcing it to pass through the origin to satisfy the restriction that $Y = 0$ for $M = 0$.

The model proposed in this paper can be regarded as a combination of both the covariance and total nutrient approaches. On one hand, the soil test variables for nutrients N (t_n) and P (t_p), are treated as any other variables in a polynomial model. On the other, a constant value is added to the levels of fertilizer nutrients N (d_n) and P (d_p) such that Eq. [4] becomes:

$$N = N_f + d_n \quad [5]$$

$$P = P_f + d_p \quad [6]$$

The values of d_n and d_p should be regarded only as constants that optimize the goodness of fit allowing for yield not to be nil at $N_f = 0$ and $P_f = 0$. They can be estimated either with a nonlinear regression procedure or through an iterative algorithm. By extrapolation, d_n and d_p correspond to the absolute values of N_f and P_f for which $Y = 0$. They should not be taken as measures of nutrient availability in the soil.

The other site-specific variables, such as rainfall from sowing to maturity and crop rotation with maize (*Zea mays* L.) or soybean [*Glycine max* (L.) Merr.] enter the model also as covariables with direct effects and relevant interactions.

The functional form proposed here to represent this model is a transcendental function of the type:

$$Y = b_0 N^{(b_1 + b_8 C + b_{10} t_n)}$$

$$\exp(b_3 N + b_4 P + b_5 NP + b_6 R + b_7 C + b_9 t_n + b_{11} t_p) \quad [7]$$

Table 1 shows a list and a brief description of the variables included in the model.

MATERIALS AND METHODS

Data

Data were obtained from 38 on-farm experiments conducted in 1981 and 1982 to assess the response of wheat (*Triticum aestivum* L.) to N and P in the Humid Pampa, Argentina (Senigaliesi et al., 1983). In these experiments, urea and triple superphosphate were broadcast applied and incorporated into the soil before sowing. Besides the fertilizer variables N_f and P_f , the following site variables (soil test, climate, and management variables) were also taken into account. The soil test variables were the initial levels of nitrate (NO₃) and Bray-1 P

Table 2. Estimated coefficients, *t*-ratios, and their significance level.

Variable name†	Estimated coefficient	<i>t</i> -ratio (441 df)	Significance
Constant	6.6801	78.8	**
lnN	0.12642	7.33	**
lnP	0.041972	1.88	NS
N	-0.000094326	-0.15	NS
P	0.0017757	0.74	NS
NP	0.000013217	0.69	NS
R	0.0024835	16.48	**
C	0.21387	4.98	**
C lnN	-0.026549	-2.37	*
<i>t_n</i>	0.0046327	5.06	**
<i>t</i> lnN	-0.00081788	-3.45	**
<i>t_p</i>	0.013317	4.72	**
<i>t_p</i> lnP	-0.0023562	-2.32	*

*, ** Significant at the 0.05 and 0.01 probability levels, respectively.
 † N = N_i + 1 and P = P_i + 1.

in 0–20 cm soil samples. The climate variable was rainfall from sowing to maturity, and the management variable was crop rotation (Table 1).

Regression Analysis

Equation [7] was log-transformed into Eq. [8], which is linear in its parameters. The latter were estimated using the ordinary least squares method:

$$\ln Y = \ln b_0 + b_1 \ln N + b_2 \ln P + b_3 N + b_4 P + b_5 NP + b_6 R + b_7 C + b_8 C \ln N + b_9 t_n + b_{10} t_n \ln N + b_{11} t_p + b_{12} t_p \ln P \quad [8]$$

The values of *d_n* and *d_p* in Eq. [5] and [6] were obtained through an iterative procedure that maximizes the coefficient of determination (*R*²) of the ordinary least squares regression. The sampling average and standard deviation for rainfall, *t_n*, and *t_p* are given in Table 1.

Economic Analysis

The economic optima are derived for relevant input/output price ratios. These ratios are computed with the working formula:

$$r_m = [P_m(1 + R)]/[P_o(1 - a)] \quad [9]$$

where *r_m* is the relevant price ratio for nutrient *M*; *P_m* is the field price of nutrient *M*, which is estimated by taking into account all the costs involved in the application of one unit of nutrient (i.e., transport and labor); *R* is an estimate of the minimum rate of return acceptable to farmers in the region, which primarily comprises the cost of capital and the return to managerial skills; *P_o* is the field price of wheat grain, which is equal to the difference between sale price and the sum of all the costs proportional to yield (i.e., costs of harvesting, threshing, and transporting grain); *a* is the downward adjustment for likely lower yield by farmers as compared with those

Table 3. Fertilizer recommendations for wheat in the Humid Pampa, Argentina. Transcendental model for different rainfall expectations, previous crop, and soil test values.†

<i>t_p</i> ‡	Rainfall from sowing to maturity = 150 mm					Rainfall from sowing to maturity = 250 mm					Rainfall from sowing to maturity = 350 mm									
	<i>N^o</i>	<i>t_n</i> §				<i>t_p</i>	<i>N^o</i>	<i>t_n</i>				<i>t_p</i>	<i>N^o</i>	<i>t_n</i>						
		0	20	40	60			80	0	20	40			60	80	0	20	40	60	80
Previous crop: maize																				
0	<i>N^o</i>	29	26	23	19	15	0	<i>N^o</i>	41	37	32	27	21	0	<i>N^o</i>	64	56	48	40	31
	<i>P^o</i>	5	5	5	5	5		<i>P^o</i>	8	8	8	9	9		<i>P^o</i>	18	18	17	17	16
5	<i>N^o</i>	30	27	24	20	16	5	<i>N^o</i>	42	37	32	27	22	5	<i>N^o</i>	62	54	46	39	31
	<i>P^o</i>	3	3	4	4	4		<i>P^o</i>	6	6	6	6	6		<i>P^o</i>	13	12	12	12	12
10	<i>N^o</i>	32	28	24	21	16	10	<i>N^o</i>	43	38	33	28	22	10	<i>N^o</i>	61	53	46	38	30
	<i>P^o</i>	2	2	2	2	2		<i>P^o</i>	3	3	4	4	4		<i>P^o</i>	7	7	7	7	7
15	<i>N^o</i>	33	29	25	21	17	15	<i>N^o</i>	44	39	34	28	23	15	<i>N^o</i>	60	52	45	38	30
	<i>P^o</i>	0	0	0	0	0		<i>P^o</i>	1	1	1	1	1		<i>P^o</i>	2	2	2	2	2
20	<i>N^o</i>	36	32	27	23	18	20	<i>N^o</i>	47	42	36	30	24	20	<i>N^o</i>	62	55	47	39	31
	<i>P^o</i>	0	0	0	0	0		<i>P^o</i>	0	0	0	0	0		<i>P^o</i>	0	0	0	0	0
Previous crop: soybean																				
0	<i>N^o</i>	26	22	18	14	10	0	<i>N^o</i>	37	31	26	20	14	0	<i>N^o</i>	61	50	40	30	20
	<i>P^o</i>	6	6	6	6	7		<i>P^o</i>	10	10	10	10	11		<i>P^o</i>	25	22	21	20	20
5	<i>N^o</i>	27	23	19	15	10	5	<i>N^o</i>	37	32	26	20	14	5	<i>N^o</i>	56	47	38	29	20
	<i>P^o</i>	4	4	4	4	5		<i>P^o</i>	7	7	7	7	8		<i>P^o</i>	16	15	15	14	14
10	<i>N^o</i>	28	24	19	15	10	10	<i>N^o</i>	38	32	26	20	14	10	<i>N^o</i>	53	45	36	28	19
	<i>P^o</i>	2	2	2	3	3		<i>P^o</i>	4	4	4	4	5		<i>P^o</i>	9	9	9	8	8
15	<i>N^o</i>	30	25	20	15	11	15	<i>N^o</i>	38	33	27	20	14	15	<i>N^o</i>	52	44	35	27	19
	<i>P^o</i>	0	0	0	0	0		<i>P^o</i>	1	1	1	1	1		<i>P^o</i>	3	3	3	2	2
20	<i>N^o</i>	31	26	22	17	11	20	<i>N^o</i>	41	36	28	22	15	20	<i>N^o</i>	53	45	37	28	19
	<i>P^o</i>	0	0	0	0	0		<i>P^o</i>	0	0	0	0	0		<i>P^o</i>	0	0	0	0	0

† Relevant price ratios *r_n* = 8 and *r_p* = 18.32.
 ‡ *t_p* = P soil test (Bray-1 P, mg kg⁻¹ in 0–20 cm soil depth).
 § *t_n* = N soil test (NO₃⁻, mg kg⁻¹ in 0–20 cm soil depth).
 ¶ *N^o* and *P^o* = N and P economic optimum, kg ha⁻¹.

of researcher-managed experiments on farmers' fields. A detailed explanation about how to estimate the relevant field prices is given in CIMMYT (1988).

For practical purposes, however, and in order to obtain results that are comparable to those of Senigagliesi et al. (1983), the price ratios used in this paper are the same as the most realistic price ratios used by those authors: i.e., $r_n = 8$ and $r_p = 18.32$; where r_n and r_p are the relevant price ratios for N and P, respectively.

The first derivatives of the response function with respect to N and P, f_n and f_p , were equated to the price ratios r_n and r_p . The optimal levels N^* and P^* were found by solving the system of simultaneous equations $f_n = r_n$ and $f_p = r_p$ using a numerical method.

RESULTS AND DISCUSSION

Table 2 shows the estimated coefficients and their corresponding t -ratios. The estimated model not only accounted for a significant percentage of the total variability in the dependent variable ($\ln Y$), with $R^2 = 0.56^{**}$ ($P = 0.01$; $n = 454$), but it also showed coefficients with signs in conformity with agronomic expectations.

The iterative estimation of d_n and d_p in Eq. [5] and [6], with a precision of 1 kg ha^{-1} , resulted in $d_n = d_p = 1 \text{ kg ha}^{-1}$.

Fertilizer recommendations for wheat derived with this function are arranged in Table 3. The following results can be highlighted: (i) The values of N^* and P^* are compatible with the range of optima obtained with per-site economic analyses (data not shown); (ii) N^* and P^* increased as rainfall increased; (iii) N^* was lower for wheat after soybean than for wheat after maize; (iv) N^* was higher for smaller values of soil N, while P^* was practically unaffected by soil N; (v) P^* was higher for smaller values of soil P, while N^* was affected by soil P, with a pattern that depended on both previous crop and rainfall; and (vi) wheat after soybean requires less P^* than wheat after maize.

In the Pampa region, the importance of previous crop, soil test NO_3^- , and other explanatory variables is shown in a separate study by Novello et al. (1986) and later emphasized by García et al. (1987).

The N^* and P^* values in Table 3 are given as examples only to show the approach used in this paper to build recommendation tables. The functional form of the transcendental function, described in Eq. [7], has derived both economically and agronomically sound fertilizer recommendations. In addition, data requirements to use the model are achievable by both on-farm research practitioners in developing countries and in regions where soil test results can readily be obtained. In cases where soil testing is not possible, the approach in this paper

can be used to obtain unbiased estimates of soil test variables. Recommendations can be derived from the sampling averages of the soil test variables and the values of the other relevant site variables.

ACKNOWLEDGMENTS

The authors thank Derek Byerlee and three anonymous reviewers of *Agronomy Journal* for their comments on earlier drafts of this paper and to Carlos Senigagliesi, who kindly provided the data used for this paper.

REFERENCES

- Anderson, J.R. 1968. A note on some difficulties in response analysis. *Aust. J. Agric. Econ.* 12(2):46-53.
- Anderson, R.L., and L.A. Nelson. 1971. Some problems in the estimation of single nutrient response functions. Institute of Statistics Mimeograph Series no. 737, Raleigh, NC.
- CIMMYT. 1988. From agronomic data to farmer recommendations: An economics training manual. Rev. ed. Mexico, D.F.
- Colwell, J.D., A.R. Suhet, and B. van Raij. 1988. Statistical procedures for developing general soil fertility models for variable regions. CSIRO Div. of Soils Divisional Rep. 93. Canberra, ACT, Australia.
- Debertin, D.L. 1986. *Agricultural production economics*. Macmillan, New York.
- de Janvry, A. 1972. The class of generalized production functions. *Am. J. Agric. Econ.* 54:234-237.
- García R., C.A. Senigagliesi, and M.A. McMahon. 1987. Fertilizer requirements and management issues for nonacid soils in nonirrigated areas. p. 212-219. In A.R. Klatt (ed.) *Wheat production constraints in tropical environments*. Proc. Int. Conf. UNDP/CIMMYT, Chiang Mai, Thailand.
- Hildreth, C. 1957. Possible models for agronomic-economic research. p. 176-186. In E.L. Baum et al. (ed.) *Economic and technical analysis of fertilizer innovations and resource use*. Iowa State College Press, Ames.
- Mombiela, F.A., J.J. Nicholaides III, and L.A. Nelson. 1981. A method to determine the appropriate mathematical form for incorporating soil test levels in fertilizer response models for recommendation purposes. *Agron. J.* 73:937-941.
- Nelson, L.A. 1987. Role of response surfaces in soil test calibration. p. 31-40. In J.R. Brown (ed.) *Soil testing: Sampling, correlation, calibration, and interpretation*. SSSA, Madison, WI.
- Nelson, L.A., R.D. Voss, and J. Pesek. 1985. Agronomic and statistical evaluation of fertilizer response. p. 53-90. In O.P. Engelstad (ed.) *Fertilizer technology and use*. 3rd ed. SSSA, Madison, WI.
- Novello, P., A. Legaza, and M. Peretti. 1986. Evaluación conjunta de la fertilización y otros factores de productividad que explican la variación del rendimiento en el cultivo de trigo. p. 332-349. In *Primer Congreso Nacional de Trigo*, Pergamino, Actas. Vol. 3. Buenos Aires, Argentina.
- Sanchez, P.A., R.K. Perrin, and S.W. Buol. 1981. Concepts of program design for soil research and information transferral in developing countries. p. 55-66. In J.A. Silva (ed.) *Experimental designs for predicting crop productivity with environmental and economic inputs for agrotechnology transfer*. Dep. Paper 49. Hawaii Inst. of Tropical Agric. and Human Resources, Univ. of Hawaii, Manoa.
- Senigagliesi, C.A., R. García, S. Meira, M.L.R. de Galetto, E. Frutos, and R. Teves. 1983. Fertilización del cultivo de trigo en el norte de la provincia de Buenos Aires y sur de Santa Fé. Informe Técnico no. 191. EERA Pergamino, INTA, Argentina.