

*Application of Risk
Decision Theory
To Net Benefit
& MRR Analysis*

Teaching Notes
Allan Low, June 1989

I. Definition of Risk	1
II. Risk Decision Theory	1
A. Subjective Probabilities	1
B. Risk Preferences	2
C. Certainty Equivalents	2
D. Farmers' Decision Criteria under Risk	3
III. Application to Technology Evaluation	3
A. Estimating Expected Outcomes	4
B. Accounting for Risk Aversity	5
1. Mean, Variance Analysis	6
2. Net Benefit Discount	7
C. Marginal Rates of Return with and without Risk Analysis	8
D. Discussion	8

I. Definition of Risk

Whenever a specific action may give different outcomes when repeated at a different time, we can say there is risk attached to that action. In other words, where we cannot predict the outcome of the action with complete certainty, we have risk.

Farming is a particularly risky activity. Climate, diseases, prices are all unpredictable and affect the outcome of any farming activity. When farmers evaluate one action (practice) against another they will take account of the outcome variability (risk) attached to each action. This teaching note summaries the theory about how people make decisions in risky situations and then suggests how this theory may be applied in the evaluation of technologies for small farmers

II. Risk Decision Theory

There are two major elements to modern decision theory under risk. The first has to do with the **PROBABILITIES** of outcome from a specific action. That is the extent of outcome variability. This is a function of a particular technology in a given environment.

The other element has to do with farmer response to variability of outcomes, often referred to as risk aversion. This is a personal characteristic of the farmer, or group of farmers and has to do with **RISK PREFERENCES** and **CERTAINTY EQUIVALENTS**.

A. Subjective Probabilities

For any action that gives variable outcomes, it will be possible for farmers to attach probabilities of obtaining different outcomes. For example a farmer's current technology set for growing maize (A) may give \$1000 net value of output 50% of the time and \$2000 the other 50% of the time.

In this case we can say that the farmers expected outcome would be:

$$\$1500 = (1000 \times .5) + (2000 \times .5)$$

more generally we can say that:

$$E = (a_1 \cdot p_1) + (a_2 \cdot p_2)$$

Where: E = the expected outcome

a₁ = outcome possibility 1

p₁ = probability of achieving outcome possibility 1

a₂ = outcome possibility 2

p₂ = probability of achieving outcome possibility 2

Farmers will base decisions about practices on their own estimates of expected outcome. Farmers' own estimates of expected outcome are based on their own experience of the variability of outcomes and this is termed SUBJECTIVE PROBABILITY.

A farmers's subjective probability estimates may differ from his neighbour's and may differ from the actual or objective probabilities of any action. This gives rise to the possibilities of improving farmer decision making by providing information on risky outcomes that allows farmers subjective probabilities to more nearly equate with the actual or objective probabilities of outcomes of a particular action.

B. Risk Preferences

Suppose we introduced an alternative set of practices (B) for growing maize which gave higher outcomes (\$5000) 50% of times, but gave a loss (-\$2000) the other 50% of times. We can calculate the expected outcomes of alternatives (A) and (B) as in the table below.

Possibilities (States)	Probabilities	Alternative Practices (A)	Alternative Practices (B)
1	.5	2000	5000
2	.5	1000	-2000
Expected outcome		1500	1500

Expected outcomes are the same for each alternative practice. But it is clear that they are not equally attractive. Whether (A) or (B) is more attractive depends on the RISK AVERSITY of the individual decision taker.

Only if the individual is risk neutral would he be indifferent to choosing between (A) or (B). If he were risk averse he would select (A). If he were a risk taker he would choose (B).

C. Certainty Equivalents

To establish whether an individual is risk averse or risk neutral we ask "What is the certain income which would make you indifferent between a risky outcome and that income?".

For example in relation to practices (A), we could ask what certain income would you just accept rather than take the risk involved with (A)?. If the answer is less than \$1500, then the individual is risk averse, since he thinks he will get less than the expected income on the risky gamble. If the answer is more than \$1500, this means that the individual hopes to get more than the expected income with the risky gamble and is a risk taker.

In effect we see that risk averse individuals discount expected outcomes, while risk takers inflate expected incomes.

D. Farmers' Decision Criteria under Risk

We can safely assume that resource poor farmers are risk averse. On this assumption, modern decision theory tells us that farmers base risky decisions on two elements:

1. Subjective probability judgements on outcomes of actions, providing estimates of expected outcome.
2. Discount levels applied to expected outcomes to equate with certainty equivalents.

To estimate expected outcomes we need probability distribution information, subjective preferably or objective otherwise.

Estimation of risk aversity discounts is more difficult. However in choosing between two alternatives it is the relative discounts that will be important and we can make some statements about situations in which risk discounts will be higher than in others. For example discounts will tend to be higher:

1. for poorer than richer farmers
2. for technologies with higher levels of variability of outcomes
3. less familiar technologies with which farmers have had little experience
4. technologies requiring higher levels of management.

On the basis of 2. and 3. above, small farmers would select technology (A) over technology (B), despite them both having the same level of expected outcome.

III. Application to Technology Evaluation

In this section we look at ways of applying decision theory ideas in selecting between technologies or evaluating new against current technologies. We will follow the steps laid out above and consider first the estimation of expected

outcomes. We will then look at incorporating risk aversity and certainty equivalent concepts into the analysis.

A. Estimating Expected Outcomes

Trials conducted over a number of years and on many different sites in a target area will provide information on the variability of treatments and enable an estimation of expected outcome to be made for each treatment.

Between year and between site variation may need to be treated differently to arrive at expected outcome. While the outcome from each site will usually be accorded equal probability, outcomes in exceptional years should be accorded lower probability than others. Information on rainfall patterns over past seasons will be required to make these judgements.

Let us take an example. The data in Table 1 represents the net benefits for groundnuts in a trial testing the effect of Single Super Phosphate (SSP) application. The farmers' current practice of no fertiliser application is compared with the application of 200kg of SSP. Let us suppose that these data have been accumulated over 2 years from 16 sites in the first year and 4 sites in the second year.

If a comparison of rainfall patterns over past years with that obtained in each of the 2 years suggests that each year's rainfall pattern could have occurred with equal probability, then each observation would receive an equal probability weighting and we could proceed with the analysis using the means for each of the treatments.

If however one of the years is exceptional, we would want to give the net benefits recorded in those years a lower probability weighting than the others. Let us assume that the rainfall pattern in the second year (sites 17-20) was exceptionally favourable, and that we would expect such conditions to occur something less than half as frequently as the more normal conditions represented by the first year. More specifically let us assume the following probabilities for the two types of years:

Type of Year	Probability	Number of years	Overall Probability
Normal	0.05625	16	0.9
exceptional	0.025	4	0.1

The overall probability from our 20 observations must total 1, which it does. However this does not mean that there is a 90% probability of getting a normal year and

only a 1% probability of experiencing an exceptionally good one. We happen to have 4 times as many normal year observations as exceptionally good ones, so if we divide the 0.9 probability by 4, to put it on an equal footing with the exceptional year, we see that normal years are 2.25 times as likely as exceptionally good ones.

In order to calculate the expected outcome from this set of data we merely have to multiply each of the observed net benefits by its appropriate probability weighting. Summing these will give us the expected outcome for each treatment.

Table 1. Observed and Probability Weighted Net Benefits:
Groundnuts with and without SSP

Site	Observed Net Benefit		Probability Weighted Net Benefit	
	0	200	0	200
	SSP	SSP	SSP	SSP
1	396.56	627.80	22.31	35.31
2	460.51	509.64	25.90	28.67
3	465.98	455.62	26.21	25.63
4	431.83	376.32	24.29	21.17
5	472.53	323.20	26.58	18.18
6	454.32	321.89	25.56	18.11
7	420.95	653.06	23.68	36.73
8	380.49	705.95	21.40	39.71
9	482.50	445.74	27.14	25.07
10	243.49	237.53	13.70	13.36
11	329.69	374.12	18.55	21.04
12	377.04	267.27	21.21	15.03
13	658.79	661.63	37.06	37.22
14	742.55	514.04	41.77	28.91
15	645.58	623.08	36.31	35.05
16	775.56	646.22	43.63	36.35
17	1508.24	2122.75	37.71	53.07
18	1903.78	1982.00	47.59	49.55
19	1886.71	2327.52	47.17	58.19
20	1743.04	2157.08	43.58	53.93
	Expected Outcome		611.33	650.28

B. Accounting for Risk Aversity

Decision theory indicates that risk averse farmers will apply higher discounts to alternatives with greater levels of variation. Estimation of the relative variability of technologies is therefore an essential

step in risk analysis. In order to obtain an estimate of the variability of a technology we need to make use of as many observations as possible, and this is why we didn't simply use a single mean for each type of year in our estimation of expected outcome.

1. MEAN, VARIANCE ANALYSIS

We can use mean, variance analysis to develop a measure of the relative variability of different technologies. The standard deviation is a measure of variability, but its magnitude is related to the size of the mean and thus, all else being equal, technologies with larger means will have larger standard deviations. A better measure of relative variability is called the INDEX OF VARIATION.

$$\text{INDEX OF VARIATION} = \frac{\text{STANDARD DEVIATION}}{\text{MEAN}}$$

From the observations in Table 1, we can calculate the index of variation for the two technologies, and we can do this for the observed net benefits as well the probability weighted net benefits.

	Observed Net Benefit		Probability Weighted Net Benefit	
	0 SSP	200 SSP	0 SSP	200 SSP
mean	739.01	816.62	30.57	32.51
standard dev	531.30	681.09	10.02	13.09
index of var	0.72	0.83	0.33	0.40

There are two things to notice from this analysis:

- the SSP treatment has a higher degree of variability than the current practice.
- the variabilities of the probability weighted net benefits are less than the observed means.

We would expect (b), since we decided that the exceptionally high net benefits would only occur half as often as the rest - i.e. we expect lower variability than we have observed.

The fact that the probability weighted net benefits are more variable for the SSP technology means that we cannot simply judge between the treatments on the basis of expected net benefits, since we expect risk averse farmers to discount the more variable SSP

treatment more than they would their current, less variable treatment.

There are two methods we can use to determine whether a treatment with a higher mean probability weighted net benefit and higher index of variation will be preferred by a risk averse farmer. Stochastic dominance analysis is one such method, but is computationally somewhat burdensome. An alternative, rougher, but computationally simpler approach is suggested here.

2. NET BENEFIT DISCOUNT

We can assume that risk averse farmers will develop certainty equivalents that discount expected outcome of alternatives with higher degrees of variability more heavily than those with lower degrees of variability.

On this assumption we might decide to apply a 5% discount to the more variable, SSP treatment, to put it on the same certainty equivalent footing as the current practice. To get to expected outcomes, we merely have to multiply the mean values of the probability weighted net benefits by the number of observations (in this case 20).

By applying an appropriate discount factor, we can go from expected outcomes to certainty equivalents. Thus we reduce the SSP treatment expected outcome by 5%, leaving the current practice expected outcome unchanged.

	Observed Net Benefit		Probability Weighted Net Benefit	
	0 SSP	200 SSP	0 SSP	200 SSP
mean expected outcome	739.01	816.62	30.57	32.51
			611.40	650.20
certainty equivalent	739.01	775.80	611.40	617.69

Selecting the appropriate differential discount to apply is clearly critical. But we can see from the following marginal rate of returns analysis, that even quite small differential discounts as applied here can make a big difference to the decision criterion. In

~~some cases even very large discounts will not change~~
 the results of a marginal rate of return analysis based on straight means or expected outcomes. But where moderate differential discounts make a significant difference, then we have to worry that the risk aspects of the technology may make it unacceptable to farmers.

C. Marginal Rates of Return with and without Risk Analysis

We can now calculate the marginal rate of return for going from the current practice to using SSP, based on certainty equivalents, rather than mean net benefits. The costs that vary in making the change from zero SSP to 200Kg/ha of SSP total to \$65.

	Zero SSP	200 SSP	Change in NB	Change in Cost	MRR %
mean values	739	817	78	65	120
expected outcome	611	650	39	65	60
certainty equivalent	611	618	7	65	11

Using mean values in this example gives us a marginal rate of return of 120%, which would generally be considered above the minimum acceptable rate of return, and therefore worth recommending.

However when we take risk into account, the use of SSP becomes more questionable.

D. Discussion

The approach suggested above does require some judgements about the relative probabilities of getting good or bad years and about the levels of discount to apply in converting expected outcomes to certainty equivalents. However these judgements are no more arbitrary than would be used in alternative safety first decision rules such as minimum net benefit analysis or marginal rate of return confidence limit analysis.

Advantages of this approach are:

1. It requires researchers to relate their results to the typicality of the season for a particular target area. Systematic recording and maintenance of trial results in relation to rainfall experiences has not been well done partly because it has not been clear how these data would be used, except to assist a year by year interpretation of results.

2. It allows the use of the same sets of data to draw conclusions about different ecological areas with different weather pattern probabilities.
3. The probability weightings and discount factors can be continually updated as more years of data become available. In fact accumulation of this information over time will allow refinement of probability estimates and simulates farmers' development of subjective probability estimations.
4. In dryland farming, where year to year rainfall variability is perhaps the major risk factor, accumulation of variability data over time can lead to more objective probability estimates and assist farmer decision making.