Ecological Modelling, 25 (1985) 81–93
Elsevier Science Publishers B.V., Amsterdam – Printed in The Netherlands

A MATHEMATICAL MODEL FOR FORMULATING INTERCROP PROPORTIONS FOR INTERCROPPING SYSTEMS' DESIGN

P.O. ADETILOYE

Department of Plant Science, University of Ife, Ile-Ife (Nigeria)

(Accepted 14 August 1984)

ABSTRACT

Adetiloye, P.O., 1985. A mathematical model for formulating intercrop proportions for intercropping systems' design. *Ecol. Modelling*, 27: 81–93.

The concept of land equivalent coefficient (LEC), formulated for the valuation of intercrop interactions and productivity in simple to complex crop mixtures, is further employed in this work to formulate intercrop proportions for intercropping research based on the concept of expected LEC. This framework is based on the assumption that the proportion of each component crop within a crop mixture is a probability statement on its expected contribution to final yield. The product of these proportions gives the predicted or expected land equivalent coefficient value which is an index of the potential productivity of the mixture.

In a mixture of N components, the smallest expected component's proportion is $1/N$, which gives the optimum sole crop population equivalent, $(1/N)N$, of a mixture. With N as the common denominator, mixtures can attain a theoretical maximum of N optimum sole crop populations and N^N possible combinations of intercrop proportions. However, the practical combinations of component proportions on an arable land is restricted to 2^N such that the maximum population pressure is twice that of a sole crop.

This model suggests that mixtures have an inbuilt tendency to give yield advantages as the number of crop components and mixture plant population increases and as intercrop proportions become more equitable. The productivity of crop mixtures can therefore be represented and predicted in mathematical terms.

INTRODUCTION

Intercropping research often involves the manipulation of the proportion of each crop species in an attempt to achieve maximum utilization of space and growth resources. A fundamental constraint to this type of research is the lack of a common and objective framework for formulating mixture proportions. This has resulted in a major divergence in methodology of various intercropping studies and even instances of wrong judgement about mixture performance. The conclusion from a poorly formulated mixture could be that mixtures are not superior to sole crops and decision may

0304-3800/85/$03.30

therefore be made that intercropping is not profitable. The resurgence of interest in intercropping research could be rewarded with rapid progress by avoiding mere accumulation of data in this subject area if a reliable framework exists for formulating intercrop proportions.

In this paper, the concept of land equivalent coefficient (LEC), originally formulated for the evaluation of intercrop interactions and the productivity of simple to complex crop mixtures (Adetiloye et al., 1983), is further employed to formulate potentially productive intercrop proportions for intercropping research based on the concept of expected LEC. In this framework, the proportion of each intercrop component is regarded as a probability statement on its contribution to final yield. The product of these proportions, which is a measure of potential intercrop interaction, gives the predicted response of crops in intercropping. It is thus possible to discard intercrop proportions that show values less than a given expected minimum response from intercropping trials while still planning an experiment.

PROBLEM OF MODEL CHOICE

Mixtures of crops have been obtained mainly in two ways: the additive model and the substitutive model, which includes the replacement series. The additive model involves intercropping optimum populations of sole crops, thus the crop populations that will go into 2 ha are compressed into 1 ha. Evans (1960) obtained crop mixtures by adding sole crop populations together. Willey and Osiru (1972) indicated that a disadvantage of this method is that the total population of mixtures is greater than that of sole crops. Thus, if each sole crop population is less than optimum, higher total yield will always be obtained due to low population pressure of sole crops, while mixture population is just adequate. They therefore used the substitutive model in which complementary ratios (1/3 + 2/3; 2/3 + 1/3 etc.) of component crops' sole crop populations were mixed such that the intercropped land area and population pressure remained the same as in sole crop. The likely problem of overestimating mixture yield can be avoided by relating mixture yield to yield of a proven optimum sole crop population.

Fisher (1977) observed poor yield (non-transgressive yielding) from trials he conducted on the additive model for maize and soybean mixture. He therefore favoured the use of the replacement method in intercropping studies. This appears too broad a generalization since inhibitive and compensatory competitions are not likely to exist in all situations where optimum sole crop populations are intercropped. This is because the relationship between mixture components depends on a number of variables which include differences in plant structure (growth habit and canopy architecture), growth cycle of mixture components, population levels in different seasons, and spatial arrangement of species. These are avenues through which substantial complementarity and increase in mixture yields can be achieved. On the last variable – that is spatial arrangement – Haynes and Sayre (1956), for instance, studied maize response to within row competition in an attempt to alter cultural practices for maize to facilitate intercropping. Maximum maize yield for intercropping was obtained from half (10 cm) of the usual within row spacing of 20 to 22.5 cm when between row spacings doubled. Thus, critical investigations are necessary to elucidate the relationship between species under specific sets of conditions.

Huxley and Miangu (1978) observed that a major disadvantage of the replacement model is that it provides for a limited number of plant populations. They therefore suggested the need for a wider range of possible mixture populations in the assessment of intercropping advantage. This is because both crop combinations and mixture plant populations are likely to exhibit substantial interaction.

The replacement method has it own agronomic advantages such as enhanced utilization of growth resources which can lead to yield increases per plant of the taller and hence dominant cereal crop (Willey and Osiru, 1972; Remison, 1978; Fisher, 1979) when compared with yield per plant in a sole crop. This not withstanding, it appears that the replacement model, which has so much emphasis in intercropping research, involves decision-making which is more of an economic than an agronomic problem. There is no doubt, however, that the process of mixing crops by farmers involves the elements of choice of species and that of making a decision on species proportion with particular reference to the expected net returns from likely harvests of individual species.

However, it appears more desirable to avoid choosing intercrop proportions on the basis of the replacement series only, since intercropping is not just a question of the ability of mixture components to substitute for one another in terms of nutritional or economic value to the farmer. More important is the efficient utilization of soil nutrients and other resources through enhanced complementary association in mixtures. This essentially is an agronomic problem which can be mathematically resolved. This does not rule out the fact that improved intercropping practices need to be accompanied with adequate socio-economic studies undertaken at various stages at the farmers level in order to relate the alternative patterns to farmers' situations and needs.

PROBLEM OF MIXTURE POPULATION CHOICE

Studies on intercropping of maize and groundnuts reported from Ghana, Cameroun and Nigeria in West Africa appear somewhat conflicting. Koli (1975) in Ghana intercropped maize and groundnuts in alternate rows, 46 cm apart and 46 cm within row or in alternate hills at 46 cm between rows and within hills. It was observed that groundnut yields were drastically reduced and was therefore suggested that sole crops of maize and groundnuts were better than mixtures of maize and groundnuts. A report from IITA (1975), however, indicated that intercropping groundnuts with maize resulted in better returns in terms of calories or cash equivalents than from sole cropping of maize. Maize sown in the furrows showed less lodging but yielded less than if grown on ridges as the groundnuts. This indicated that both spatial arrangement and land preparation also affected crop performance.

Mutsaers (1978) in Cameroun observed a 6% yield advantage during the short rains of September to December and 16% yield advantage during the long rains of March to July from a maize–groundnuts mixture. A replacement method which involved one-third maize + two-thirds groundnuts and vice-versa was used. When an optimum population of groundnuts was intercropped with 1/3 or 2/3 of optimum population of maize, the land equivalent ratio was 1.4 or 40% yield advantage over the sole crop. It was observed that mixing groundnuts with a low maize plant population reduced the risk of supra-optimal maize population where nitrogen was a limiting factor since maize was able to exploit nitrogen from a greater soil volume.

Baker (1978) in Nigeria also observed greater cash returns from cereals and groundnuts mixtures than from a sole crop of groundnuts. Intercropping maize in addition to sorghum and millet was observed to have increased cash returns even further. Groundnuts were planted at 100 cm × 23 cm while the cereals were planted interchangeably at 100 cm × 137 cm or 100 cm × 274 cm in the same row as groundnuts. The low productivity of a maize-groundnuts mixture earlier reported by Koli (1975) was probably due to the relatively high maize population used such that the closeness of maize to groundnuts did not make for substantial spatial complementarity between the two crops. Willey (1979), noted that greater canopy differences can lead to greater light utilization. It would appear that the critical factor in the case of the maize-groundnuts mixture (Koli, 1975) is the height of maize.

Mixtures involving optimum populations of two or more component species have not been widely studied. Willey and Osiru (1972) also indicated that little information is available on the additive model which they considered an important aspect of intercropping. The problems of mixture formulation with respect to the choice of model, components proportions and mixture population are being tackled in the present paper.

MODEL OF INTERCROP PROPORTIONS AND MIXTURE PLANT POPULATION

The ratios of 1 : 1 and 1 : 1 : 1 which are the optimum sole crop population equivalents are common treatments in studies on two and three crops mixtures, respectively. The use of other ratios, for example two-thirds of A and one-third of B (2 : 1) or vice-versa in two-crop mixtures, appear rather too arbitrary. The proportion of intercrops constitute a probability statement on their expected contribution to final yield. The proportions of intercrops are additive assuming no interaction occurs. The product of intercrop proportions for the sole equivalent is on the other hand regarded in this study as the expected minimum productivity level for the mixture (Table I). The product of the relative contributions in terms of yield by each crop component is called the land equivalent coefficient (LEC) while the product of intercrop proportions is called the expected land equivalent coefficient. The use of the LEC concept for the evaluation of mixture productivity and inter-specific interaction had been discussed (Adetiloye et al., 1983).

The idea of expected minimum productivity level is presently extended to establish the logical proportions of component crops that can yield above this expected minimum. The distribution of these proportions are presented in Tables II and III for mixtures of two and three crops, respectively. The use of the above proportions which are projected from optimum sole crop levels constitute a reliable principle for choosing intercrop proportions in intercropping design and for evaluating component crops performance or interaction.

Since, however, a crop mixture usually consists of components of different competitive abilities, it must be noted that the following theoretical or mathematical equalities encountered within a probability distribution are different treatments in the actual intercropping situation (Table IV).

TABLE I

Expected minimum and maximum intercrop proportions for crop mixtures

Number of crop components	Sole crop population equivalent ratios	Minimum expected values		Theoretical maximum mixture proportions	Maximum expected values		
		LER	LEC		LER	LEC	PC
2	1:1	1.0	$(1/2)^2$	2:2	2	1^2	10^2
3	1:1:1	1.0	$(1/3)^3$	3:3:3	3	1^3	10^3
4	1:1:1:1	1.0	$(1/4)^4$	4:4:4:4	4	1^4	10^4
5	1:1:1:1:1	1.0	$(1/5)^5$	5:5:5:5:5	5	1^5	10^5
N	$1/N:1/N: \ldots :1/N$	1.0	$(1/N)^N$	$N:N: \ldots :N$	N	1^N	10^N

LER, mixture land equivalent ratio.
PC, mixture productivity coefficient.
LEC, land equivalent coefficient.

TABLE II

Distribution of expected mixture proportions and the corresponding expected land equivalent ratio (LER), land equivalent coefficient (LEC), and the productivity coefficient (PC) values for mixtures of two crops

Expected proportions of component crops		Distribution of mixture proportions	Expected values		
A	B		Mixture LER (A+B)	Mixture LEC (A×B)	Mixture PC (LEC×10²) (%)
½	½	$P_1^aP_1^b$	1.00	0.25	25
½	1	$P_1^aP_2^b$	1.50	0.50	50
1	½	$P_2^aP_1^b$	1.50	0.50	50
1	1	$P_2^aP_2^b$	2.00	1.00	100

Here P_1, P_2 and P_3 are intercrop proportions, while superscripts a, b and c denote the different intercrop species. The four intercrop combinations in $(P_1 + P_2)^2$ for mixtures of two crops therefore constitute four different treatments rather than three suggested by the expansion coefficients of 1 : 2 : 1. This is because while P_1 in $P_1^aP_2^b$ represents the proportion of the first mixture component, P_1 in $P_2^aP_1^b$ represents the proportion of the second mixture component, thus actual interspecific effects would differ between $P_1^aP_2^b$ and $P_2^aP_1^b$. Similarly, there are 27 different treatments for $(P_1 + P_2 + P_3)^3$ rather than ten indicated by the expansion coefficients. A uniform spatial arrangement or cropping pattern is thus assumed when using the above distribution of proportions. If for instance, two spatial arrangements are being considered for N mixture components, the number of treatments will double.

As a result of intercrop interaction, mixture yields usually deviate from the expected. There is therefore the need to experiment with mixtures of different crop components and at different levels of complexity under various cropping environments so as to determine the magnitude of deviation from the theoretical. Intercropping instead of planting sole crops can be advantageous due to more efficient utilization of available space caused by differences in plant structure, canopy architecture and maturity dates. The greater the number of crops the more likely the variability in canopy characteristics and the differences in resource utilization that can satisfy the requirements of intercrop components and the less likely is competition for space. Mixtures with expected land equivalent ratio (LER) of 1.00 which are numbered "a" (Tables II and III) are therefore, likely to give greater yield advantages where the number of crop components are more (Table III). The

TABLE III

Distribution of expected mixture proportions and the corresponding expected land equivalent ratio (LER), land equivalent coefficient (LEC), and the productivity coefficient (PC) values for mixtures of three crops

Expected proportions of component crops			Distribution of mixture proportions	Expected values		
				Mixture LER	Mixture LEC	Mixture PC
A	B	C		(A+B+C)	(A×B×C)	(LEC×10³) (‰)
1/3	1/3	1/3	$P_1^aP_1^bP_1^c$	1.00a	0.037	37
1/3	1/3	2/3	$P_1^aP_1^bP_2^c$	1.33	0.074	74
1/3	1/3	1	$P_1^aP_1^bP_3^c$	1.66	0.111	111
1/3	2/3	1/3	$P_1^aP_2^bP_1^c$	1.33	0.074	74
1/3	2/3	2/3	$P_1^aP_2^bP_2^c$	1.66	0.148	148
1/3	2/3	1	$P_1^aP_2^bP_3^c$	2.00b	0.222	222
1/3	1	1/3	$P_1^aP_3^bP_1^c$	1.66	0.111	110
1/3	1	2/3	$P_1^aP_3^bP_2^c$	2.00b	0.222	222
1/3	1	1	$P_1^aP_3^bP_3^c$	2.33	0.333	333
2/3	1/3	1/3	$P_2^aP_1^bP_1^c$	1.33	0.222	222
2/3	1/3	2/3	$P_2^aP_1^bP_2^c$	1.66	0.148	148
2/3	1/3	1	$P_2^aP_1^bP_3^c$	2.00b	0.222	222
2/3	2/3	1/3	$P_2^aP_2^bP_1^c$	1.66	0.148	148
2/3	2/3	2/3	$P_2^aP_2^bP_2^c$	2.00b	0.295	295
2/3	2/3	1	$P_2^aP_2^bP_3^c$	2.33	0.444	444
2/3	1	1/3	$P_2^aP_3^bP_1^c$	2.00b	0.222	222
2/3	1	2/3	$P_2^aP_3^bP_2^c$	2.33	0.444	444
2/3	1	1	$P_2^aP_3^bP_3^c$	2.66	0.666	666
1	1/3	1/3	$P_3^aP_1^bP_1^c$	1.66	0.111	111
1	1/3	2/3	$P_3^aP_1^bP_2^c$	2.00b	0.222	222
1	1/3	1	$P_3^aP_1^bP_3^c$	2.33	0.333	333
1	2/3	1/3	$P_3^aP_2^bP_1^c$	2.00b	0.222	222
1	2/3	2/3	$P_3^aP_2^bP_2^c$	2.33	0.444	444
1	2/3	1	$P_3^aP_2^bP_3^c$	2.66	0.666	666
1	1	1/3	$P_3^aP_3^bP_1^c$	2.33	0.333	333
1	1	2/3	$P_3^aP_3^bP_2^c$	2.66	0.666	666
1	1	1	$P_3^aP_3^bP_3^c$	3.00c	1.000	1000

TABLE IV

Effect of different spatial arrangements of intercrop proportions on the mathematical model for formulating mixtures

Mathematics	Intercropping situation
$P_1P_2 = P_2P_1$	$P_1^aP_2^b \neq P_2^aP_1^b$
$P_1P_2P_3 = P_1P_3P_2 = P_3P_2P_1$	$P_1^aP_2^bP_3^c \neq P_1^aP_3^bP_2^c \neq P_3^aP_2^bP_1^c$

TABLE V

Ideal and practicable distributions of mixture proportions for intercropping systems' design

No. of mixture components	Expected mixture proportions					Maximum number of combinations of proportions	No. of mixtures	Practicable proportions for inter-cropping	No. of mixtures
	P_1	P_2	P_3	P_4	P_5				
2	$\frac{1}{2}$	1	–	–	–	$(P_1 + P_2)^2$	4	$(P_1 + P_2)^2$	4
3	1/3	2/3	1	–	–	$(P_1 + P_2 + P_3)^3$	27	$(P_1 + P_2)^3$	8
4	$\frac{1}{4}$	2/4	3/4	1	–	$(P_1 + P_2 + P_3 + P_4)^4$	256	$(P_1 + P_2)^4$	16
5	1/5	2/5	3/5	4/5	1	$(P_1 + P_2 + P_3 + P_4 + P_5)^5$	3125	$(P_1 + P_2)^5$	32
N	$1/N$		…		N/N	$(P_1 + P_2 \ldots + P_N)^N$ $\sum_{i_1=1}^{n} \sum_{i_2=1}^{n} \ldots \sum_{i_n=1}^{n} P_{i_1} P_{i_2} \ldots P_{i_n}$	N^N	$(P_1 + P_2)^N$ $\sum_{i_1=1}^{n} \sum_{i_2=1}^{n} P_{i_1} P_{i_2}$ or $\sum_{k=0}^{n} \binom{n}{k} P_1^k P_2^{n-k}$	2^N

same is true for mixtures numbered "b" with expected LER of 2.00, as the number of component crops increases since variation in growth requirements will be greater the more the number of mixture components. It appears, however that mixture productivities at "a" or at "b" cannot readily be compared for different levels of mixture complexity (in terms of number of intercrops) when the LEC is used to evaluate mixture performance. While the LER showed linear or arithmetic increases, the LEC showed geometric increases in mixture productivity as mixture population increased. The more rapid rate of increase observed for the LEC values suggests synergistic interaction and not additive association (Tables I, II and III). The superiority of the LEC over the mixture LER is therefore more apparent as the number of component crops increases assuming any increase in the number of mixture components will correspondingly increase mixture plant population and yield.

The fundamental questions to be answered are: What is the maximum number of components that a mixture can accommodate?; and What is the optimum population pressure for a mixture since theoretically productivity increases with number of components and at higher population pressures? There is no logical limit to the number of crops that can go into the formulation of a mixture since greater canopy differences and maturity dates can both confer increasing advantages on mixtures, provided other things (soil fertility and climatic conditions) are equal. Mixtures could be designed to contain higher plant population than sole crops because the growth resources available to each crop is higher in mixtures than with equal plant population of a sole crop. The extent to which such additional resources (due to more space) can be exploited is limited by the two-dimensional form of a cropping field in which the seeds are arranged in the soil. In some yam growing areas of eastern Nigeria, however, mounds of about 1 m high and 3 m base diameter are planted to ten yam setts and other crops per mound instead of one yam sett usually used per small mound possibly to obtain higher yield per unit land area based on a third-dimensional advantage. It is therefore desirable to investigate the practicable number of crops in a mixture and the extent to which mixture population could be increased above the optimum sole crop population level under different cultural practices, rainfall and light regimes.

It is possible to intercrop up to twice the optimum sole crop population of a number of arable crops due to differences in canopy characteristics or by varying the planting dates of components involving selected genotypes and by choosing the best spatial arrangement. The ideal and practicable distributions of expected intercrop proportions for crop mixtures is summarized in Table V. The various intercrop proportions outlined in Table V for agronomic studies on intercropping systems (up to twice the optimum sole crop population) include both the substitutive and the additive models. Thus this resolves the limitations earlier observed in the replacement series by Huxley and Miangu (1978) and Willey and Rao (1981). Whether a substitutive or additive model is used would depend on the prevailing level of growth factors and the nature of interaction among component crops in a given cropping pattern. Thus, there is the need to determine crop performance in various cropping patterns in relation to the levels of various growth factors. The effects of differences in plant height, growth habit and leaf area index of component crops must also be determined.

Table VI indicates two different cases where the product of intercrop proportions can be used to resolve the problem of making a choice between or among possible alternative combinations. The element of choice comes in when a farmer or researcher wants to decide which of these possible alternatives is likely to confer greater yield advantage on the mixture. Case I is illustrated with mixtures of two, three and four crops. Case II is illustrated with mixtures of two and four crops. Case I shows mixtures with the same expected LER value but with different expected LEC values. In Case I mixtures, it was found that the expected LEC value will normally decrease where disparity is greater between or among the intercrop proportions. This framework eliminates probable mixtures of poor performance with small expected LEC while an experiment is being planned. For example, mixtures of 1/3 of A plus 2/3 of B and 1/4 of A plus 3/4 of B will give expected LEC values of 0.22 and 0.19, respectively. These values are less than the

TABLE VI

Effects of component proportions on interspecific interaction and expected mixture productivity

Number of component crops (N)	Component		Proportions		Expected values		
	A	B	C	D	Mixture LER (Sum A+B+ ...)	Mixture LEC (Product A×B× ...)	Mixture PC (LEC × 10^N)
2 (Case I)	1/2	1/2			1.00	0.25	LEC × 10^2
	3/4	1/4			1.00	0.19	LEC × 10^2
2 (Case II)	1/2	1/2			1.00	0.25	LEC × 10^2
	1	1/4			1.25	0.25	LEC × 10^2
3 (Case I)	2/3	2/3	2/3		2.00	0.296	LEC × 10^3
	1/3	2/3	1		2.00	0.222	LEC × 10^3
4 (Case I)	1/2	1/2	1/2	3/4	2.25	0.0938	LEC × 10^4
	1/4	1/2	3/4	1/4	2.25	0.0703	LEC × 10^4
	1/4	1/2	1/2	1	2.25	0.0625	LEC × 10^4
4 (Case II)	1/2	1/2	1/2	3/4	2.25	0.0938	LEC × 10^4
	1/4	1/2	3/4	1	2.50	0.0938	LEC × 10^4

expected minimum productivity level of 0.25 for a mixture of two crops. In the mixtures cited, the proportion of the B component is close to that for optimum sole crop population. In such a case, mixture yield is likely to fall below expectation if component crop B should compete with A. Willey and Rao (1981), using a systematic design, observed from chickpea/safflower mixtures that LER's at 2 : 1 ratio indicated no evidence of yield advantages from intercropping while at 1 : 1 ratio, yield advantage reached 19%. This result is in agreement with the prediction of this model. This is also an advantage of the LEC concept over the LER concept.

The lower LEC value obtained for mixtures with marked disparity in their components' proportions illustrates the situation in natural communities. In natural communities, the poor competitive ability of the less represented species confer competitive advantage on species that are more in number and are more vigorous. This results in progressive elimination of the less favoured species (interspecific thinning) in the course of each successsion. Where, however, the proportion and vigour of several species are more equitable, the system is more stable and productive.

In Case II (Table VI), the component proportions are different and so are the expected LER values while the expected LEC values are the same. In this case, it would be better to choose a mixture with the smaller LER value since seed requirements per unit land are less while the outcome of the interaction effects are expected to be similar for both intercrop combinations. A close similarity in the formulation of the two-crop mixtures and that of four-crop mixtures is also worthy of note in Table VI.

The distribution of intercrop proportions yielded by this framework is not a probability distribution and does not therefore conform to the notion of additive probability in which the sum of the probabilities of N events is one (Table V). However, this framework reasonably agrees with the Cantorian set theory which was reviewed by Dauben (1983). The potential productivitiy of crop mixtures when planted at mixture populations greater than that of a sole crop equivalent is constrained by the two-dimensional character of the cropping field. This therefore imposes a limitation on the ideals of this set theory. This is why the combinations of intercrop proportions for field evaluation are limited to $(1/N + 2/N)^N$. The combinations contain a minimum of sole crop population equivalent and a maximum equivalent of double the sole crop population where N is equal to two or more intercrops.

In using this model, the sole crop comparison for each intercrop should be included as treatments in each intercropping experiment for accurate determination of competition effects. The sole crop comparison for the calculation of land equivalent coefficient is the yield from an optimum sole crop population for the same land area that is occupied by N mixture components.

SUMMARY AND CONCLUSIONS

The land equivalent coefficient (LEC) concept, developed to evaluate competition and productivity in crop mixtures, has been further employed to formulate the proportion of intercrops that can perform at or above a minimum expected threshold for positive interspecific interaction. The expected LEC value can then be compared with the actual LEC values of mixtures for different cropping environments and practices in order to determine the intercrop proportions or treatments that show worthwhile mixture yield advantages.

For mixtures to be meaningfully systematised, mixtures should be planted in ratios that use the number of crops in the mixture (N) as a common denominator such that the reciprocal of N, $1/N$, will be the smallest possible intercrop proportion for each crop component. The maximum and practicable combinations of intercrop proportions that this model can yield are N^N and 2^N, respectively where N is the number of mixture components.

The expected productivity values calculated for LEC indicated that mixtures have an inbuilt tendency to give yield advantages as intercrop proportions become more equitable and as the number of crop components and the population pressure increases. The additive and the substitutive models included in this mathematical model should therefore be included as treatments in intercropping trials. This would provide a better understanding of the actual basis for higher mixture yields. Statistical methods for evaluating the performance of crop mixtures at different population levels based on this model will be covered in future papers. The choice of intercrop proportions permitted by this model can also satisfy the economic interests of farmers. Various possible planting arrangements should also be tried in order to establish the best practice under a given set of circumstances. The present method of choosing intercrop proportions also makes it possible to simultaneously include the study of other factors such as plant type effects, fertilizer rates or plant arrangement without the number of component crops increasing the number of factors unduly. If any such factor was to be considered for each intercrop, however, the design would become extremely complicated.

REFERENCES

Adetiloye, P.O., Ezedinma, F.O.C. and Okigbo, B.N., 1983. A land equivalent coefficient (LEC) concept for the evaluation of competitive and productive interactions in simple to complex crop mixtures. Ecol. Modelling, 19(1): 27–39.

Baker, E.F.I., 1978. Mixed cropping in Northern Nigeria. I. Cereals and Groundnuts. Exp. Agric., 14: 293–298.

Dauben, J.W., 1983. Georg Cantor and the origins of transfinite set theory. Sci. Am., 248 (6): 112–121.

Donald, C.M., 1963. Competition among crop and pasture plants. Adv. Agron., 15: 1–118.

Evans, A.C., 1960. Studies in intercropping. I. Maize or sorghum with groundnuts. East Afr. Agric. For. J., 26: 1–10.

Fisher, N.M., 1977. Studies in mixed cropping. II. Population pressures in maize–bean mixtures. Exp. Agric., 13: 185–191.

Fisher, N.M., 1979. Studies in mixed cropping. III. Further results with maize–bean mixtures. Exp. Agric., 15: 49–58.

Haynes, J.L. and Sayre, D.J., 1956. Response of corn to within row competition. Agron. J., 49: 362–364.

Huxley, P.A., and Miangu, Z., 1978. Use of a systematic spacing design as an aid to the study of intercropping: Some general considerations. Exp. Agric., 14: 49–56.

IITA, 1975. Annual Report. International Institute of Tropical Agriculture, Ibadan, Nigeria.

Koli, E.S., 1975. Pure cropping and mixed cropping of maize and groundnuts in Ghana. Ghana J. Agric. Sci., 8: 23–30.

Mutsaers, H.J.W., 1978. Mixed cropping experiment with maize and groundnuts. Neth. J. Agric. Sci., 26: 344–353.

Remison, S.U., 1978. Neighbour effects between maize and cowpea at various levels of N and P. Exp. Agric., 14: 205–212.

Willey, R.W., 1979. Intercropping—its importance and research needs. Part I, Competition and yield advantages. Field Crop Abstr., 32(1): 1–10.

Willey, R.W. and Osiru, D.S.O., 1972. Studies on mixtures of maize and beans (*Phaseolus vulgaris*) with particular reference to plant population. J. Agric. Sci., Camb., 79: 517–529.

Willey, R.W. and Rao, M.R., 1981. A systematic design to examine effect of plant population and spatial arrangement on chickpea/safflower. Exp. Agric., 17: 63–73.